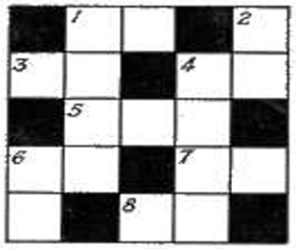


JUNIOR CROSS-FIGURE No. 22

ACROSS :

1. Width, in centimetres, of star S, if width of star S^1 is 6 m.m. (Fig. 2).
3. Average of " 6 across " and " 7 across ".
4. Exterior angle of a regular pentadecagon.
5. Area, in square centimetres, of star S^1, if area of star S is ·52 sq. cm. (Fig. 2).
6. A prime number.
7. H.C.F. of 329 and 517.
8. Perimeter, in inches, of the box (Fig. 1).

DOWN :

1. Area, in square inches, of base of the box not covered by money. Take $\pi = 3 \cdot 142$ (Fig. 1).
2. Value of x if $3(x+3)-5(x-4)=1$.
4. Number of pounds in a score of hundredweights.
6. Length, in inches, of the diagonal of the box (Fig. 1).

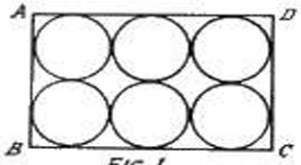


FIG. 1

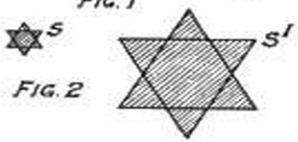


FIG. 2

Check clue : The sum of all the digits inserted is three-fifths of a century. I.H.

ABCD is a rectangular box which just holds six half-pennies arranged as shown.
S is a small star and S^1 is the star magnified 4 times.

SOLUTIONS TO PROBLEMS IN ISSUE No. 23

FOOD FOR THOUGHT

The clue is the breadroll, which must also be paid for. By adding 1 roll to each of the alternatives given on the menu, you will have four equations in four unknowns. Solving these will give the cost of a fried egg and the included breadroll as 11d.

FROM AN ANCIENT BOOK

If x hours of the day have gone, then there must be $\frac{5}{7}$ x hours left. Since these must total 24 hours, the equation becomes $\frac{5}{7}x + x = 24$, therefore $10\frac{5}{7}$ hours of the day have gone.

SENIOR CROSS-FIGURE No. 23

ACROSS : (1) 121 ; (3) 5·95 sq. cm. ; (5) £63 ; (6) 60° ; (7) 3·40 cm. ; (10) 399 ; (11) £116 ; (13) 36° ; (14) 16 cm. ; (16) 6·88 sq. cm. ; (17) 42·4.
DOWN : (1) 19 ; (2) 16036 ; (4) 9·14 ; (7) 39364 ; (8) ·01 ; (9) 31 ; (12) 108° ; (15) 54.

CHARLIE COOK

$\frac{3}{5}+\frac{1}{4}+\frac{1}{3}=\frac{47}{60}$, therefore Charlie needed 60 inkwells before he could find the correct fractions although only the original 47 were actually required for the form rooms.

ROMAN CROSS FIGURE

ACROSS : I. MDC ; III. LX ; V. VI ; VII. XIV.
DOWN : I. MIV ; II. CL ; IV. XXV ; VI. IX.

THOUGHT READING

Let n be the number chosen. By multiplying by ten and subtracting the original number, the result is 9n. Adding any multiple of 9 will make a number which is also a multiple of 9 and therefore its digits will add up to a multiple of 9. The missing digit must then be the difference between the sum of the digits given and the next highest multiple of 9.

GOING FOR A SAIL

Each ship from New York will meet all the ships which leave Southampton during the following 144 hours, *i.e.*, 6 ships. In addition, it will also meet the ship leaving Southampton at the time of its own departure from New York and the 6 ships which left Southampton during the preceding 144 hours, making a total of 13 ships in all.

JUNIOR CROSS-FIGURE No. 21

ACROSS : (2) 2·54 ; (5) ·1125 ; (7) 34 ; (8) 541 ; (10) 261 ; (12) 154 ; (13) 225 ; (15) 25 min. ; (16) ·0667 ; (19) 145°.
DOWN : (1) 31 ; (2) 22° ; (3) 555 ; (4) 14 miles ; (6) 15625 ; (7) 31536 ; (9) 41 ; (11) 12 ; (14) 504 sq. ml. ; (15) 28 ; (17) 65 miles ; (18) 79°. I.L.C.

188

68683 86894 27741 55991 85592 52459 53959 43104

MATHEMATICAL PIE

No. 24 *Editorial Offices :* 97 Chequer Road, Doncaster **MAY, 1958**

MATHEMATICAL INSTRUMENTS No. 4

The Geometric Square and the Quadrant

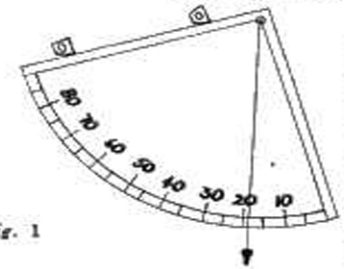


Fig. 1

The quadrant was a surveying instrument, originating in Greece, for measuring angles of elevation. It reached Western Europe at about the beginning of the XIIIth century. At its simplest, it consisted of a quarter circle of wood or metal, with an arc graduated from 0° to 90°. (*Fig.* 1). The upper straight edge carried sights, which were directed at the object whose angle of elevation was required. From a peg at the centre of the arc hung a plumb line, whose intersection with the degree scale showed the angle through which the sights had been turned from the horizontal.

The geometric square was a device for reading directly the tangent or cotangent of an angle of elevation. It consisted of a frame, two or three feet square, with equally divided scales, called shadow scales, on two of its sides. (*Fig.* 2). It might have sights and a plumb line as in the quadrant described above, but more commonly it had a plumb line on one edge, so that the frame could be held vertically, and a sighting bar, called an alidade, pivoted to one corner of the frame. The intersection of the alidade with the shadow scale showed the tangent or cotangent of the angle of elevation, according as the angle was greater or less than 45°.

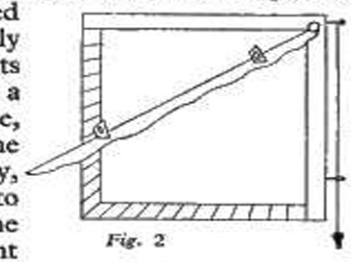


Fig. 2

It was an obvious step to combine quadrant and square in one instrument. In the plumb-line type, the shadow scales were usually drawn inside the quadrant, whilst in alidade instruments the quadrant was usually fitted inside the square.

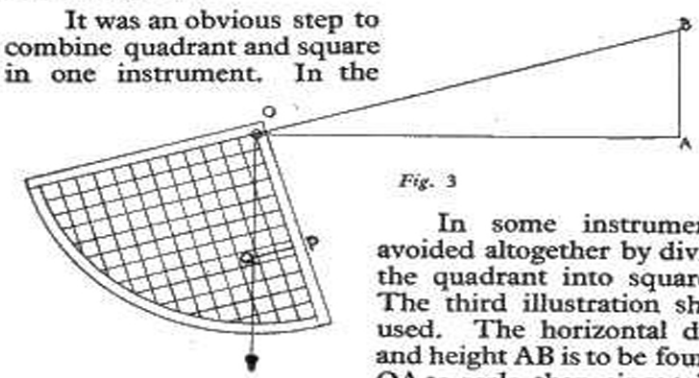


Fig. 3

In some instruments, calculation was avoided altogether by dividing the space inside the quadrant into squares, like graph paper. The third illustration shows how these were used. The horizontal distance OA is known, and height AB is to be found. If OP represents OA to scale, then, since triangles OAB and OPQ are similar, PQ represents AB to the same scale.

181

04712 37137 86960 95636 43719 17287 46776 46575

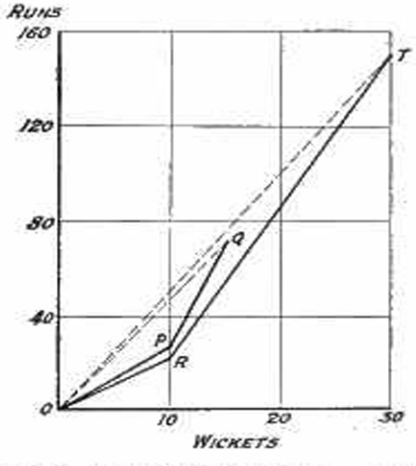
For the same degree of accuracy, a quadrant was a much smaller instrument than an astrolabe. Many large instruments were made for astronomical use. C.V.G.

MYSTIFYING CRICKET AVERAGES

Contributed by R. L. Bolt, M.Sc., Woodhouse Grove School, Nr. Bradford.

Colonel Watchem, an enthusiastic supporter of a village cricket team, had offered a prize for the player with the best bowling average at the end of the season. Just before the last match he found that the two best bowlers were level, having each taken 28 wickets for 60 runs. In the last match Hurricane Harry took 4 wickets for 36 runs, an average of 9 runs per wicket, and Whirlwind Willy took only 1 wicket for 27 runs. Before congratulating Hurricane and presenting him with the prize, the colonel thought he ought to work out the final averages. To his surprise he found that both bowlers had the same average : Hurricane had taken 32 wickets for 96 runs, an average of 3, while Whirlwind had taken 29 wickets for 87 runs, also an average of 3.

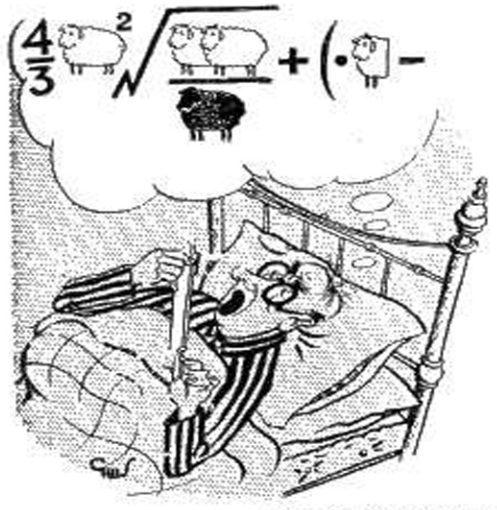
Very puzzled, the colonel decided to investigate the score book further. There were two other good bowlers in the team, Grubber Graham and Yorker Jim. He found that in the two months of May and June, Grubber had taken 10 for 26, an average of 2·6, while Yorker had taken 10 for 22, a better average of 2·2 ; in the months of July and August, Grubber had taken 5 for 44, an average of 8·8, while Yorker had taken 20 for 128, again a better average of 6·4. Thus for both halves of the season Yorker had a *better* result than Grubber. But over the whole season Grubber had taken 15 for 70, an average of $4\frac{2}{3}$, while Yorker had taken 30 for 150, a *poorer* average of 5 !



Now completely mystified he turned to the team's scorer, " Einstein " Illingworth, a sixth-former at the local grammar school and a boy reputed to have quite a brain for Maths. and asked him for an explanation.

" Einstein " started by explaining that bowling averages were really rates. " We can understand this bowling situation better if we think first of average speeds ", he said, " Supposing you walk 2 miles in 1 hour and I walk 6 miles in 2 hours. Your average walking speed is 2 m.p.h. and mine is 3 m.p.h. Next you cycle 14 miles in 2 hours and I cycle 8 miles in 1 hour. Your average cycling speed is 7 m.p.h. and mine is 8 m.p.h. I cycle faster and walk faster. But when we combine the walking and cycling, we find that you cover 16 miles in 3 hours at an average speed of $5\frac{1}{3}$ m.p.h., while I cover 14 miles in 3 hours at a *lower* average speed of $4\frac{2}{3}$ m.p.h. This unexpected result occurs because I walk for a greater part of my total time than you do yours. Now the same sort of thing happens with the bowling averages for Grubber and Yorker. Runs take the place of miles and wickets take the place of hours. Yorker kept up his high average of 6·4 for two-thirds of his wickets and his low average of 2·2 for only one-third ; Grubber on the other hand kept up his high average of 8·8 for just one-third of his wickets and his low average of 2·6 for two-thirds. Thus,

73962 41389 08658 32645 99581 33904 78027 59009



Drawn for "600" by Smith

MOVING IN THE BEST CIRCLES

Alan, Bill and Chris set off simultaneously from the same point on a circular track 3 miles in circumference. Alan cycles at 14 m.p.h., Bill runs at 8 m.p.h. and Chris walks at 5 m.p.h., all in the same direction. How long will it be before they all come together again at some point on the track? J.F.H.

FOOTNOTES FOR HISTORIANS

Warren designed the first iron and steel truss bridge in England ; built over the Trent at Newark in 1851. (Earlier bridges in cast iron were designed like masonry bridges) The joints were always hinged in earlier steel bridges, but, in 1881, the German engineer, Mandeiln, published a method of calculating the stresses introduced by rigid joints. Bridges built since are usually rivetted.

The correct shapes for masonry arches with uniform loads were calculated by the Scottish mathematician Gregory in 1697. The Italian engineer Poleni used experimental methods in 1748 for finding the correct shapes for uneven loads. The graphical methods used nowadays were devised by Karl Culman (1821–1881), Professor of Engineering at Zurich Polytechnic. Coulomb (1736–1806) investigated the stability of arches which differed from the theoretically correct shapes. The bending stresses of three hinged and two hinged arches were first investigated by Louis Naires (1785–1836).

The first cantilever bridge was built by the Bavarian engineer H. Gerber in 1867. The first in Britain was the Forth Bridge, commenced in 1883.

Concrete was patented by an English mason, Joseph Aspdin, in 1824. A boat of reinforced concrete was built in 1854 but the general use of the material began with Joseph Monier, a gardener, who made reinforced concrete flower pots in 1867 and later designed water tanks and reservoirs. C.V.G.

PEDALLING PACES

Johnny and his sister went for a long cycle ride. By way of showing his superiority, Johnny maintained a speed of 8 m.p.h. but his sister could only manage a steady 6 m.p.h. After riding for 5 hours, Johnny began to feel guilty, so he turned back. How far from their starting point would the two meet? J.F.H.

41059 78859 59772 97549 89301 61753 92846 81382

FALLACY No. 24

Contributed by J. Krabbendam, The Hague.

If $N > n$, $\log N > \log n$

$\frac{1}{2} > \frac{1}{4}$; $\therefore \log \frac{1}{2} > \log \frac{1}{4}$; $\therefore \log \frac{1}{2} > \log (\frac{1}{4})^2$, $\therefore \log \frac{1}{2} > 2 (\log \frac{1}{2})$.

$\therefore 1 > 2$!

I WANT TO BE—No. 4

An Electrical Engineer

The first impression conveyed by the above title might be that of someone carrying pliers, screwdriver and insulating tape. It is necessary, of course, that a professional electrical engineer should know all about the practical side of his business and bench work is, therefore, an important part of his training. It is also necessary that he should have a good mathematical background.

Electricity is not easily defined in simple terms, but whatever it is, it can be accurately measured in terms of its pressure, or *voltage*, and its current flow, or *amperage*. In the study of the flow of electricity through conductors, or its utilisation in the operation of apparatus ranging from a hand torch to giant rolling mill motors rated in thousands of horse power, many different quantities besides voltage and amperage have to be taken into account. Some of these, such as resistance and wattage, you may already have studied in the physics laboratory. Others, such as inductance, capacitance and impedance, may sound strange, but are important in electrical theory.

If electricity were to be supplied only by batteries and used exclusively for lighting filament lamps, the calculations involved would be quite simple. In practice however, transmission of electricity has to be made over great distances and, in order to avoid losses, pressures measured in hundreds of thousands of volts are used. For this reason, alternating current—easily transformed from one voltage to another—is generally used for long distance transmission.

Alternating current varies periodically both in voltage and amperage and the amplitude of either, plotted against time, traces out an approximation to a sine wave that repeats every fiftieth of a second at the standard British frequency. The sine waves of voltage and amperage, Fig. 2, may not always be in step with one another, so the computation of power in the circuit becomes a real mathematical problem, but at the same time a simple one compared with some of the problems with which electrical engineers may be confronted.

Some readers may have heard of i, the square-root of —1, and perhaps wondered if it was of any practical use. It so happens that the electrical engineer makes considerable use of this imaginary quantity in alternating current calculations, but renames it the "j-operator" because he already uses i as a symbol for current.

Electrical engineering offers wide scope for boys and girls interested in mathematics. The large electrical firms have probably the most comprehensive training schemes to be found in any industry. The courses of instruction include practical work in the plant and theory at a college or university. You can obtain more information by writing to the chief education officer of one of the well-known electrical firms.

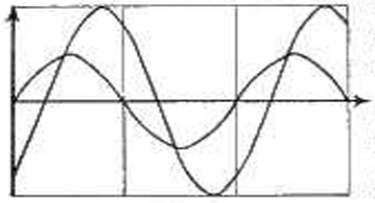


Fig. 2

J.F.H.

14294 09190 65925 09372 21696 46151 57098 58387

although Grubber's two averages were individually higher than Yorker's, his overall average was lower. Does this make things clear?"

"I am beginning to get the idea", said the colonel, "but there still seems to be something very peculiar about bowling averages. Anyway, I suppose I shall have to give two prizes this year".

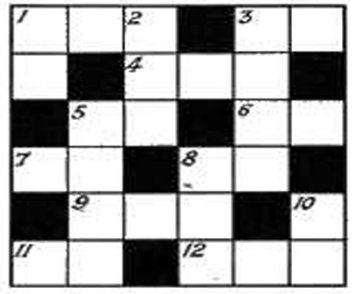
Average problems such as the above two can be illustrated graphically. Take the case of Grubber and Yorker. In the figure, P represents the wickets taken by Grubber for 26 runs. The gradient of OP represents the average of $\frac{26}{10}$, *i.e.*, 2·6 runs per wicket. Similarly, gradient PQ represents Grubber's second average of 8·8, while gradient OQ represents his overall average of $4\frac{2}{3}$. OR, RT, OT represent Yorker's averages of 2·2, 6·4, 5. It is easy to see how it is possible to have grad. OP > grad .OR and grad. PQ > grad. RT and yet find grad. O < grad. OT.

You could illustrate Hurricane's and Whirlwind's averages in the same way. You could also try making up problems of this kind yourself by first drawing diagrams on graph paper.

PUZZLING PAIRS

Twelve tickets numbered from 1 to 12 were folded and shaken up in a box. They were then drawn out in pairs and the total of the two numbers for each pair were called out. If the totals were 4, 6, 13, 14, 20, 21, what were the numbers of each pair?

J.F.H.



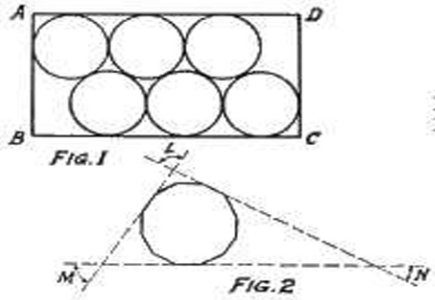


FIG. 1

FIG. 2

ABCD is a rectangular box which just holds six halfpennies arranged as shown.

Sides of a threepenny piece are produced to meet at L, M and N.

SENIOR CROSS-FIGURE No. 24

ACROSS:

1. Height, in inches, of a pint bottle of milk, assuming milk bottles are similar and that a quart bottle is 11·5 in. in height
3. Product of the roots of the equation $2y^2 + 19y + 24 = 0$.
4. Perimeter, in inches, of the box (Fig. 1).
5. $a^{\frac{3}{2}}b^2$ (see 7 across).
6. Distance, in inches, between the centres of the halfpennies that are furthest apart (Fig. 1).
7. $a + b$ if $a - 7b = 4$ and $3a - 8b = 51$.
8. The reciprocal of ·5.
9. Length RS, in centimetres, given triangle PQR with PQ = 6 cm., S the mid-point of PQ, $\angle P = \angle L$ and $\angle Q = \angle N$ (see Fig. 2 for L and N).
11. Angle M (Fig. 2).
12. Average speed, in feet per second, of a car that covers "3 across" miles in 4·8 minutes.

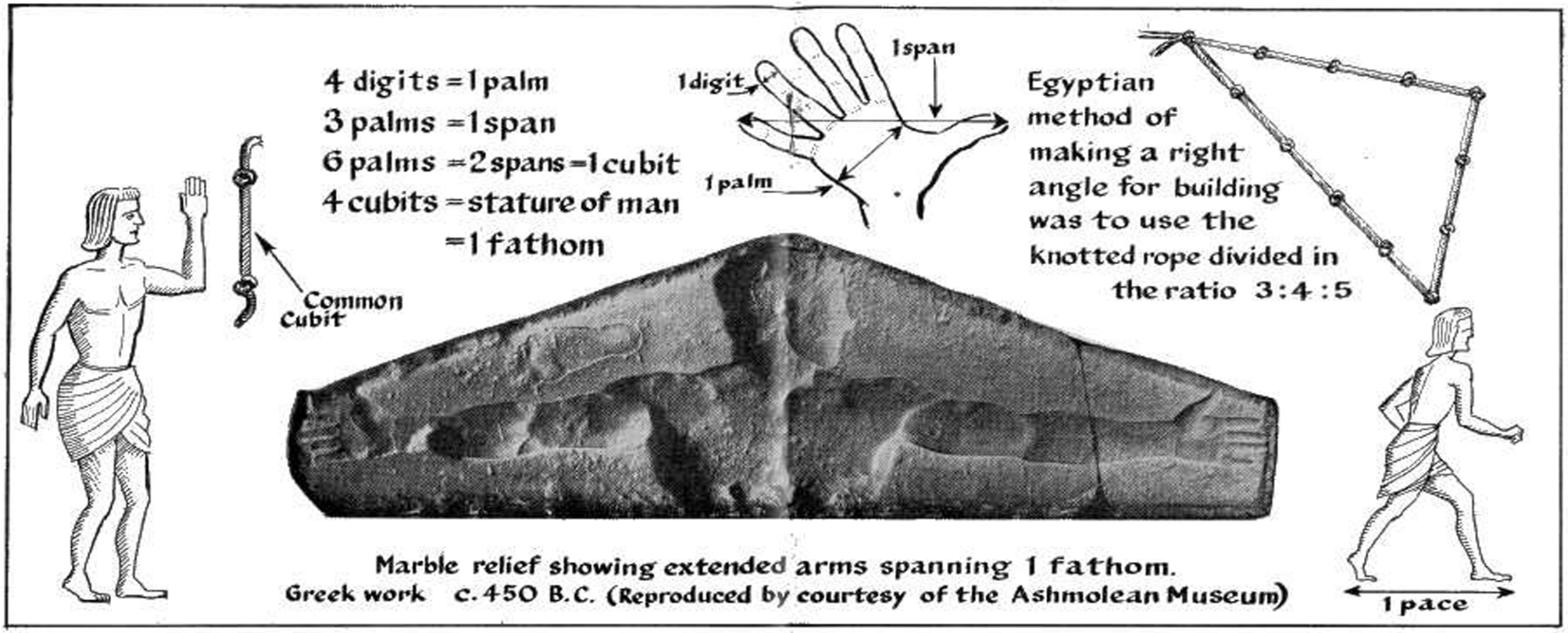
DOWN:

1. $(7\frac{1}{2} + 3\frac{3}{4} \times \frac{1}{2}) \div (\frac{3}{4} - \frac{1}{2})$.
2. The difference of the squares of 2 numbers whose product is 54 and whose quotient is 6.
3. Number of 1 inch cubes that may be cut from a 1 foot cube.
5. Square yards in a muddled race.
8. Area, in square nches, of base of the box not covered by money. Take $\pi = 3 \cdot 142$ (Fig. 1).
10. Value of c when $(3c - 1)(c + 1) + (1 - c)(3c + 2) = \frac{1}{3}$ of "1 down".

Check clue: The difference between the sums of odd and even digits inserted is 17.

I.H.

94657 64078 95126 94683 98352 59570 98258 22620



Marble relief showing extended arms spanning 1 fathom.
Greek work c. 450 B.C. (Reproduced by courtesy of the Ashmolean Museum)

Our knowledge of the methods of measurement used by people of the early civilizations has been acquired by piecing together what evidence could be obtained from archeological finds and ancient writings. However, it is not always easy to find exact values for early units in terms of our own measurements, so much of our assessment of such units can only be approximate.

Some of the oldest records tell us how the Egyptians developed a system of measurement from the need to measure the amount of land farmed by each peasant on the banks of the Nile. Originally this was done by pacing, that is by taking even strides from a marked starting point and counting the number of strides required. The ancient Greeks used a similar system in which one *pace* was a double step and 125 paces were equal to one *stadium*, which represented the distance a runner could sprint. It has been calculated that 1 stadium would be approximately equal to 607 English feet.

It was an obvious beginning that man should use his own body as a standard for measuring shorter lengths, though this did give a rather variable set of units. One of the fundamental lengths was the distance from the elbow to the tip of the longest finger, which gave a length of approximately 18 inches and was called the *common cubit of man*.

In order to facilitate the use of this measure a length of rope was knotted at 1 cubit intervals. The Egyptian surveyors who used this system were

called *harpedonaptae*, or "rope-stretchers" and mention of them is found in the Rhind Papyrus, from which we have learned much of the mathematical achievements of the early Egyptians.

A longer length than the cubit was the distance between the tips of the longest fingers when the arms were stretched horizontally on either side of the body. This was called the *stature of man* and is the length which we now take as a fathom. By using the knotted rope this new length was seen to be equal to 4 cubits.

As the idea of measurement developed it became necessary to define shorter lengths, so a further set of units was obtained by using the hand. By stretching out the fingers and thumb as far as possible and measuring the distance from the tip of the little finger to the tip of the thumb they obtained a length which became known as a *span*. Other measurements from the hand were the *palm* and *digit*.

As the Egyptians came to build greater and more impressive temples the need to standardise their units became obvious and it was thus that the Royal Cubit was taken as a standard. This, strangely enough, was based on a Babylonian measure rather than on the Egyptian common cubit and would be approximately equal to 20·64 inches. I.L.C.