EARLY DIVISION

```
STAGE 1        STAGE 2        STAGE 3
17978          17978          17978
               12             12
                 5978           5978
                 21             21
                 3878           3878
                    6              6
                 3818           3818
                                32
                                  618
                                  56
                                    58
                                    16
                                    42
472            472            472
                 472            472
                                  472
  0              03             038
```

In the sixteenth century, division was a feat which could only be performed by a skilled mathematician. The method favoured by the Arabs and Persians (800—1400 A.D.) is shown in the example 17978 ÷ 472.

472 is larger than 179, so a 0 is put in the 2 column for the answer. 472 is now written in the next three columns ; 4 times 3 is 12, 7 times 3 is 21 and 2 times 3 is 6, these are written in the appropriate columns, and subtracted in turn. The 472 is then moved one column to the right and the process is repeated. Hence 17978 ÷ 472 = 38 remainder 42. It will be noticed that the part you usually do in your head is all written down. B.A.

CLOCK ARITHMETIC No. 5

Powers and Roots

In the last issue you learned that $\sqrt{1}$ has *six* answers in modulo 6 arithmetic. If we now extend the idea of powers and roots, we begin by listing the results that $1^2=1$, $2^2=4$, $3^2=3$, $4^2=4$, $5^2=1$, $6^2=6$.

Then

$$1^3 = 1^2 \times 1 = 1 \times 1 = 1 \qquad 2^3 = 1^2 \times 2 = 4 \times 2 = 2 \qquad 3^3 = 3^2 \times 3 = 3 \times 3 = 3$$

$$4^3 = 4^2 \times 4 = 4 \times 4 = 4 \qquad 5^3 = 5^2 \times 5 = 1 \times 5 = 5 \qquad 6^3 = 6^2 \times 6 = 6 \times 6 = 6$$

In turn, this reveals that in modulo 6 arithmetic

$\sqrt[3]{1}=1$, $\sqrt[3]{2}=2$, $\sqrt[3]{3}=3$, $\sqrt[3]{4}=4$, $\sqrt[3]{5}=5$, $\sqrt[3]{6}=6$.

Are there any other answers for each of these cube roots?

Now use this method to complete the following table of powers for modulo 6 arithmetic

Number	Square	Cube	Fourth	Fifth	Sixth
1	1	1			
2	4	2			
3	3	3			
4	4	4			
5	1	5			
6	6	6			

What conclusions can you draw about a^{2n+1}? R.H.C.

82446 25759 16333 03910 72253 83748 18214 08835

MATHEMATICAL PIE

No. 46 Editorial Address: 100, Burman Rd., Shirley, Solihull, Warwicks, England OCTOBER, 1965

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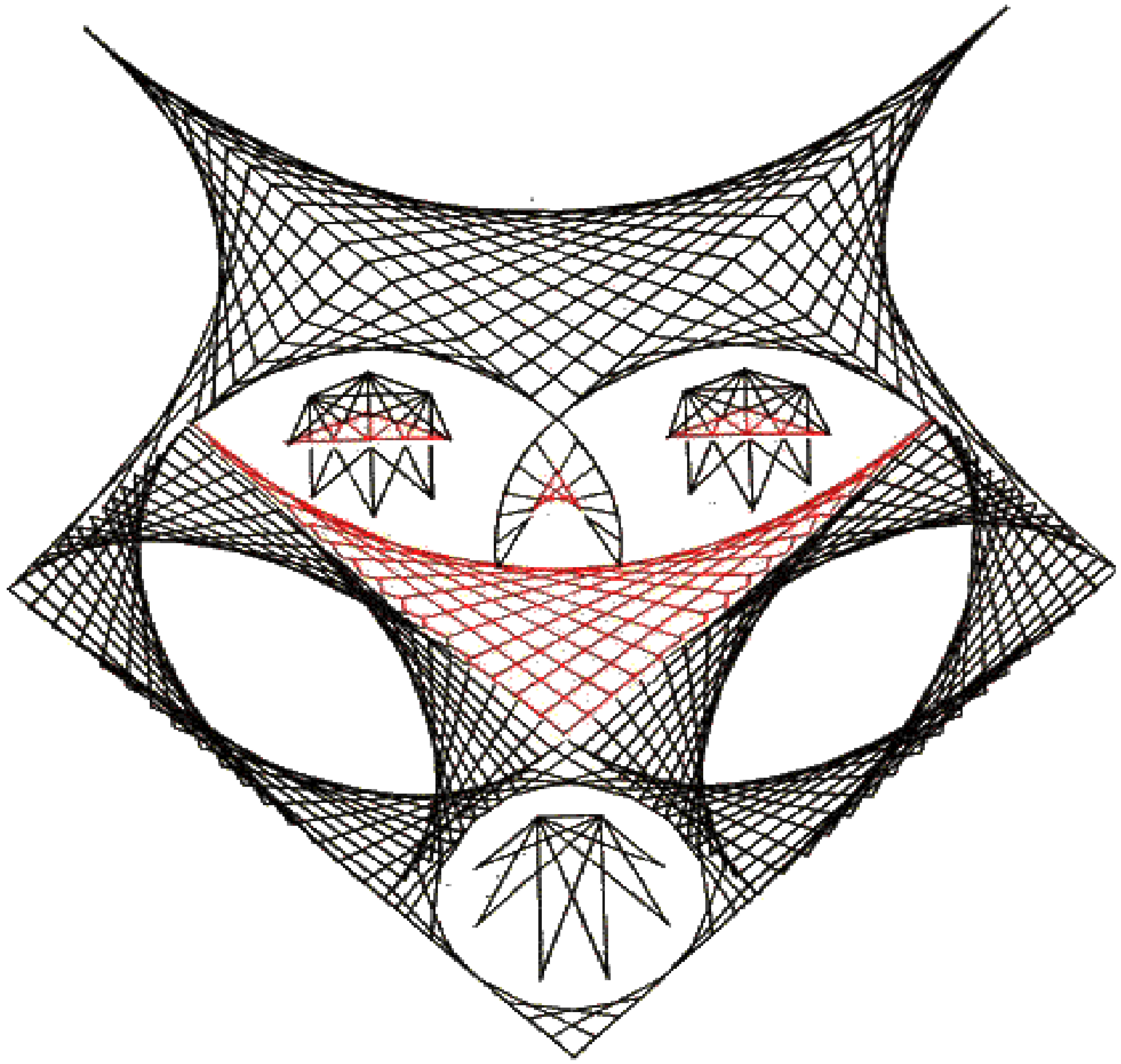
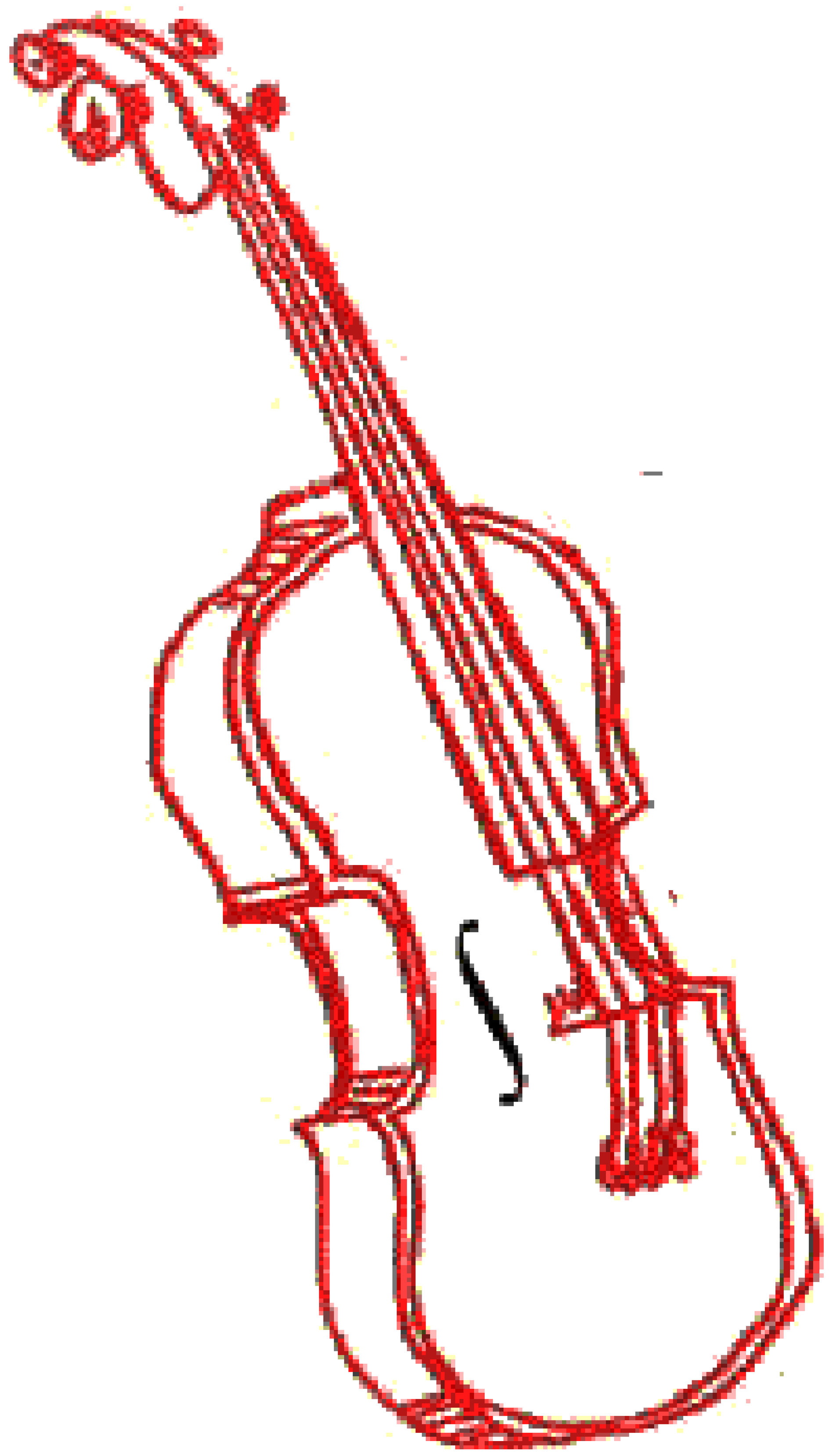


Submitted by J. R. Fox, The City School, Lincoln

65583 43434 76933 85781 71138 64558 73678 12301

THE ORIGIN OF $\int$ — MATHEMATICAL FIDDLING?

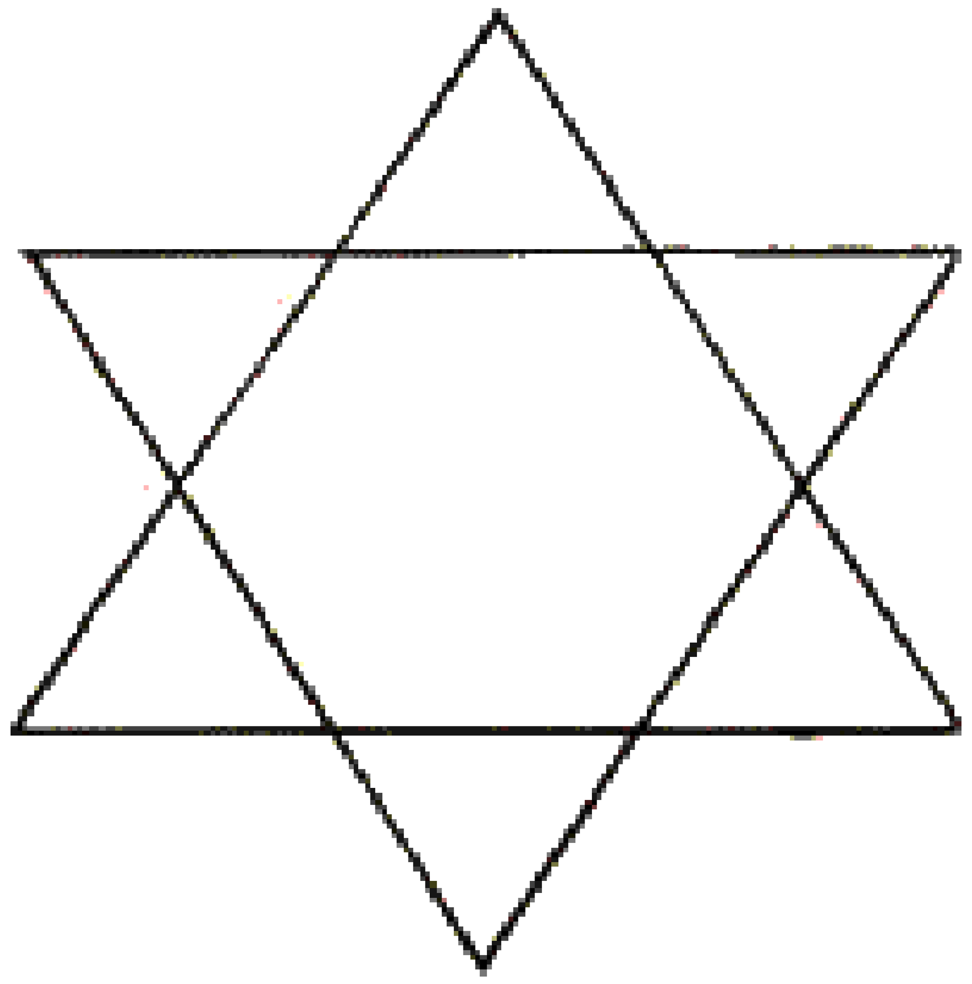
The beginner in algebra can be compared to a traveller making a cross-country walk. Every now and then, the traveller has to exert himself to climb over a stile. The first 'stile' that must be overcome in algebra is that of learning to calculate with letters instead of figures. There is no reason why figures should not be used in some algebra but, if long numbers are involved, a lot of time would be wasted in just writing: the use of symbols, therefore, is a time-saver.



The other symbols such as $+$, $-$, $\times$, and $\div$ should already be well known to the beginner because it is customary to teach arithmetic before algebra. The next stile, therefore is probably the use of brackets, to be followed in due course by other (and maybe higher) stiles such as equations, indices, factorisation, and so on. The beginner will not meet any new symbols for the next two or three years until he is one day faced with $\int$.

If the beginner becomes an engineer or a physicist, he will use $\int$ many times until he regards it as an old friend. It is a symbol first used by the celebrated Leipzig mathematician, Gottfried Wilhelm Leibniz (1646–1716) in a manuscript dated 29th October, 1675. The significance of $\int$ is that a large numb^er of very small quantities placed behind the sign are to be added together according to certain rules to provide what is called an *integral*. The integral is not always easy to find and sometimes a little 'inspired guesswork' is helpful. For this reason non-mathematicians may think of the process as 'fiddling.'

J.F.H.



STAR DISSECTION

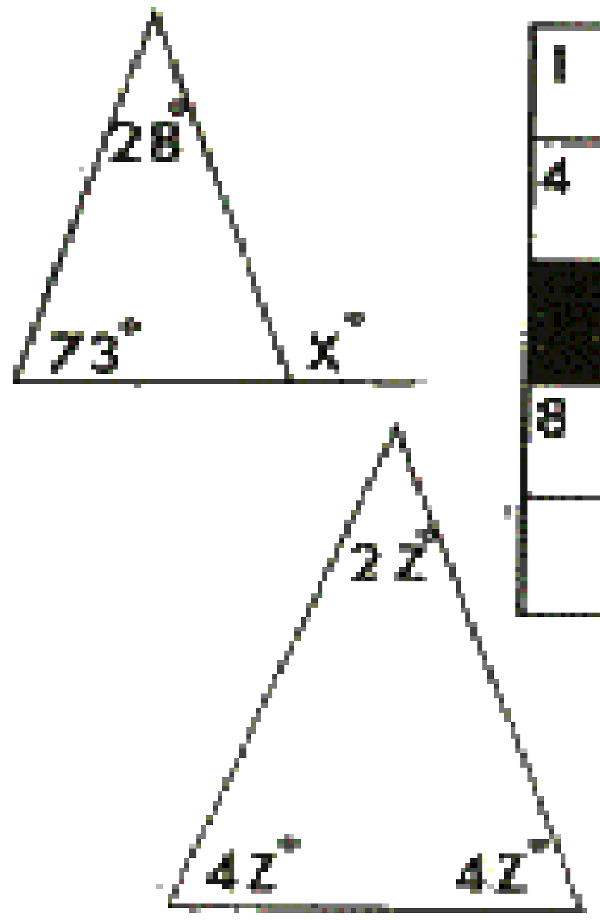
Submitted by Canon D. P. Eperson.

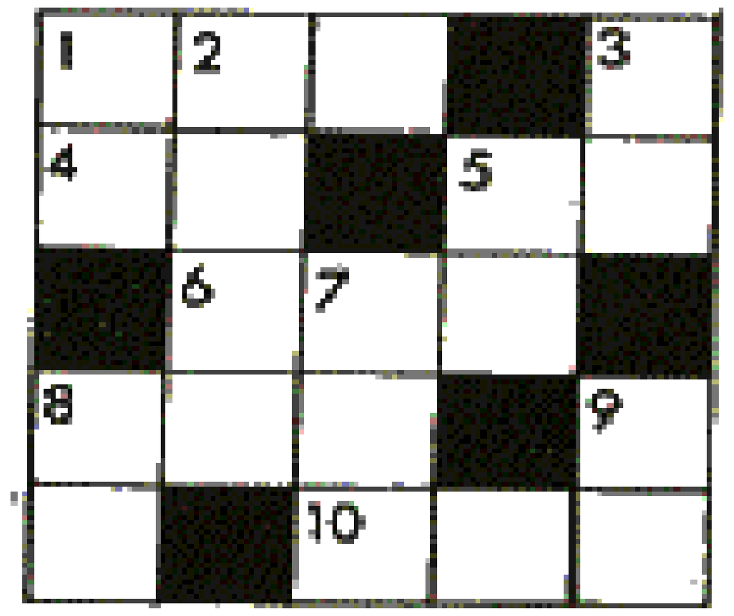
A regular hexagon can be arranged in a square by cutting it into five pieces. A hexagonal star is more complicated but it is surprising that this can be re-arranged to form a square by cutting it into five pieces.

Can you find the cuts that must be made?

45876 87126 60348 91390 95620 09939 36103 10291

JUNIOR CROSSFIGURE No. 39





CLUES ACROSS:

1. A perfect square.
4. $2p+3=61$, find p.
5. Average of $(63-y)$, 20, $(y+1)$.
6. Consecutive digits.
8. A multiple of 41.
10. Find x.

CLUES DOWN:

1. 2^5.
2. $7 \times 10^3 - 1$.
3. Find z.
5. Evaluate $3t^2+5t+5$ when $t=2$.
7. a^2-b^2 when $a=40$ and $b=27$.
8. Correct 62.4 to two significant figures.
9. $\sqrt{121}$.

B.A.

ENCIRCLING MOVEMENT

Three sides of a cyclic quadrilateral are 3", 4", 5". The angle between the first two sides is 120°. What is the length of the remaining side? J.G.

SOLUTIONS TO ISSUE No. 45

TWO FOR A PENNY

1. 503 acres. 2. $\frac{1}{4}(2\sqrt{3}-\pi)$. 3. $1\frac{1}{2}$ inches.

ALGEBRA CORNER No. 3

1. $x^2+y^2+z^2-2xy+2yz+2zx$. 2. Using the result of part 1, the sum of the edges $=\sqrt{(6^2+64)}=10$ inches. 3. $c^2=ab$ if a and b are not both zero. 4. $a=5$, $b=2$. 5. The area is $2a(a^2+1)$ square inches. 6. The angle between the fingers is $\frac{1}{2}(60y-11x)$ or $360-\frac{1}{2}(60y-11x)$.

AT THE CROSS WAYS

The solution will appear in the next issue.

THE ROARING FORTIES

I = 1, F = 2, S = 3, X = 4, E = 5, Y = 6, R = 7, T = 8, O = 9, N = 0.

SENIOR CROSSFIGURE No. 43

CLUES ACROSS: (1) 175, (3) 425, (5) 429, (7) 52, (9) 25, (10) 36288, (11) 84, (12) 65, (14) 143, (16) 714, (17) 121.

CLUES DOWN: (1) 105, (2) 54, (3) 49, (4) 525, (6) 28284, (8) 234, (9) 286, (11) 857, (13) 571, (14) 14, (15) 31.

The clue to 13 Down should have read [illegible].

PROBLEM CORNER

Printer's Error.—No other set of four digits with this property is known.

Enclosed. The greatest area is a circle, $40{,}000/\pi$ square yards. The square of area 10,000 square yards encloses the greatest rectangular area.

It is not cricket

12! = 479,001,600 and not 579,001,600 as stated.

HANDICAPPED

The girl touched the wall $5\sqrt{3}$ yards from C. The distance covered was $20\sqrt{7}$ yards. It has been suggested that she should be promoted to the Sixth.

JUNIOR CROSSFIGURE No. 38

CLUES ACROSS: (1) 2228, (4) 884, (6) 36288, (8) 568, (9) 1414.
CLUES DOWN: (1) 2835, (2) 28284, (3) 888, (5) 4884, (7) 661.

B.A.

03375 11173 54719 18530 46449 02636 55128 16228

EXTENDING THE MULTIPLICATION TABLES

Submitted by W. J. Davies, Greenhill Grammar School, Tenby.

Robert Recorde, in his "Grounde of Artes," explained how to multiply by a number between 5 and 10 because he did not advise his readers to learn the tables for multiplying beyond five, in the early stages of Arithmetic.

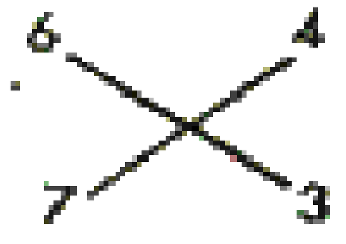


Fig. 1

To multiply 6 by 7, place the 6 and the 7 at the left hand corners of a cross, as in figure 1, subtract 6 and 7 from 10 and place these results at the right hand corners. Multiply 4 by 3 giving 12. The 2 is put in the units column and the 1 is carried into the tens column. Take 3 from 6 (or take 4 from 7), and place the result 3 in the tens column making 4 with the 1 that was carried. The final answer is 4 tens and 2 units, or 42.

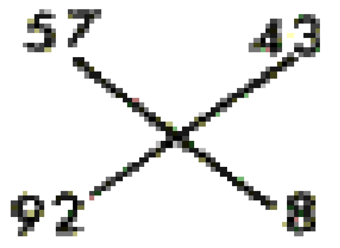


Fig. 2

It is interesting to note that this method may be extended to the multiplication of numbers between 50 and 100 or 500 and 1,000, etc. Thus to multiply 57 by 92, we could proceed as follows: $43 \times 8 = 344$. This time we put down 44 and carry 3 into the hundreds column. $57 - 8$ (or $92 - 43$) $= 49$, to which the 3 must be added to give 52. The answer is therefore 5244.

This extension of Recorde's method was "discovered" by a Form IIIA at the Greenhill Grammar School, Tenby. Can you prove why it works?

6	+		−		= 7
×	■	×	■	−	■
	+		+		= 2
+	■	+	■	+	■
	×		+		= 1
= 9	■	= 6	■	= 7	■

WITHOUT A WORD

Each empty square requires one figure so that the working from top to bottom and from left to right is correct. B.A.

A STRIKING PROBLEM

If a clock takes six seconds to strike six, how long does it take to strike (1) eleven, and (2) twelve?

A BREAKFAST TIME PROBLEM

What is the quickest way to toast three slices of bread on both sides, using a double-sided electric toaster?

BULL'S EYE

Ten concentric circles are drawn, so that the areas between two consecutive circles are all equal to the area of the inside circle. The radius of the inner circle is 1 inch, what is the radius of the outer circle? J.G.

UPWARDS EVER UPWARDS

A cube of six inch edge stands on a table. An insect crawls from one of the lower corners to the middle of the top face, but cannot climb at an angle greater than 30° with the horizontal. What is the least length of its journey? J.G.

33467 68514 22342 77379 30375 87034 43661 99106

FOR EXPERIENCED MATHEMATICIANS

It is a great pleasure for us to know that *Mathematical Pie* is read with interest by many mature readers. The following problem is directed mainly towards such readers.

Three circles, radii 3", 4", 5" touch each other. They are enclosed by the external common tangents forming a triangle. It is required to calculate the lengths of the sides of this triangle.

Younger readers will find it interesting to solve the problem by scale drawing. A book token will be given for the best solutions. J.G.

STEPPING IT UP!

Find a value for x that satisfies the equations (a), (b), (c), (d) :—

$$\text{(a) } x^{2^3} = 64, \quad \text{(b) } 2^{x^3} = 64, \quad \text{(c) } (2^3)^x = 64, \quad \text{(d) } 2^{(3^x)} = 64$$

Now find a value for y that satisfies (e), (f), (g) :—

$$\text{(e) } y^{2^3} = 4096, \quad \text{(f) } 2^2 . 2^{2^y} = 4096, \quad \text{(g) } 2 . 2^{2^{2^y}} = 4096.$$

J.F.H.

Hip-hip-hoo-rah. Hip-hip-hoo-ray,
A tin of fruit for tea today!
First the label round you'll see:
In size it's π times h times d.
Next the contents you will try—
Of course they're r^2 h times π.

PASSING BY

Two trains of equal length pass each other on parallel tracks each travelling at 50 miles per hour. An observer on one of the trains finds that it takes the other 4 seconds to pass him. How long are the trains? R.H.C.

SENIOR CROSSFIGURE No. 44

1		2	■	3
	■	4	5	
■	6	■		■
7		■	8	9
10			■	

Clues Across:

1. $\frac{3}{.01} + \frac{1.1}{.02}$
4. Convert 624.8 ft. per sec. to miles per hour.
7. Area in sq. in. of a trapezium with parallel sides 7" and 4", 6" apart.
8. Find the shortest side of a right-angled triangle, two of whose sides are 35" and 28".
10. Taking $\pi = 3\frac{1}{7}$, find the volume of a cylinder of radius $3\frac{1}{2}$" and height 6".

Clues Down:

1. If $a = 2$ and $b = -3$, evaluate $a^3 - b^3$.
2. Find x when $\frac{x+2}{7} - \frac{x-6}{8} = 2$.
3. The simple interest on £360 for 4 years at $2\frac{1}{2}$ per cent. per annum.
5. Find the width, in yards, of a rectangular field of 22 acres which is ¼ mile long.
6. $7x$ where $3x + 4y = -11$ and $5x + 6y = -7$.
7. The smallest angle of a triangle whose angles are $8x°$, $17x°$ and $20x°$.
9. The larger root of the equation $2x^2 - 21x - 36 = 0$.

B.A.

62615 28813 84379 09904 23074 73363 94804 57593

MATHEMATICAL EMBROIDERY

The centre pages of issue No. 25 were devoted to Mathematical Embroidery and such interest was aroused that the issue was quickly sold out. So many requests have been made for copies that the basic ideas are repeated with new illustrations. We are indebted to Mrs. B. J. Atkins and last year's IIB form of the Newland High School, Hull, for the basis of the examples.

Many interesting patterns may be produced by stitching to order on card or more substantial material. Holes are made, usually along a straight line or a circle, to a rule; they may be equally spaced but this is not essential. The holes are then joined to a pre-determined pattern with thread. The final design is usually attractive but can often be improved by using blending or contrasting colours of thread for different parts.

Figures 1, 3, 7, 8 and 9 are all based on parabolas which are the easiest curves to produce. Two lines are drawn and the same number of equally spaced holes are made along each. The first hole of one line is sewn to the last hole of the second line, the second hole of the first line is sewn to the next to the last hole of the second line, and so on until each hole on one line is sewn to one hole of the second line. The " envelope " of the " stitches " is a parabola, (a curve like $y = x^2$), as sixth formers may be able to prove mathematically.

Figures 4, 5 and 6 consist of holes on circles sewn in various ways which are obvious. Figure 2 consists of three " curves of pursuit." These are the paths, (*i.e.*, loci) followed by three dogs each starting from a vertex of a triangle and running directly towards the next dog taken in order around the triangle.

Chainstitch may be used to pick out lines and various types of " filling in " stitch may be used to produce areas, which can be worked into patterns. The standard text on the subject is one published in 1906 by Miss E. L. Somervell, *A Rhythmic Approach to Mathematics*, but a simplified explanation can be found in " *Curve Stitching* " by C. Birtwistle of the Association of Teachers of Mathematics. B.A.

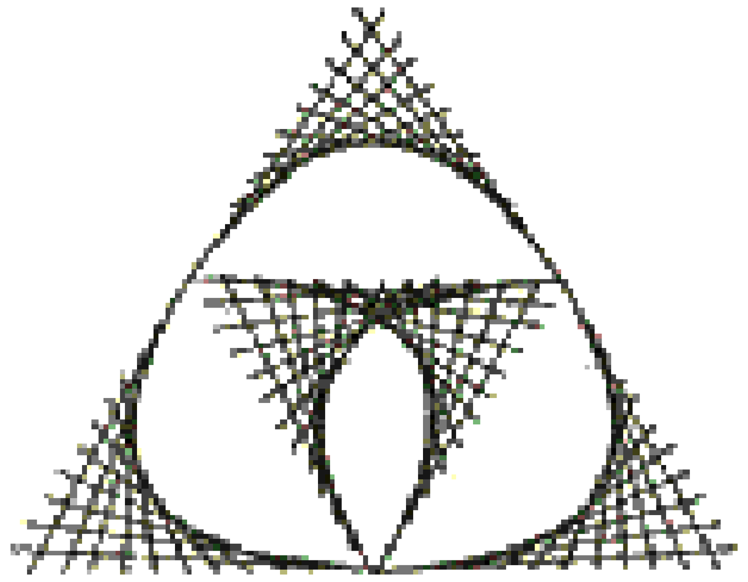


Fig. 1

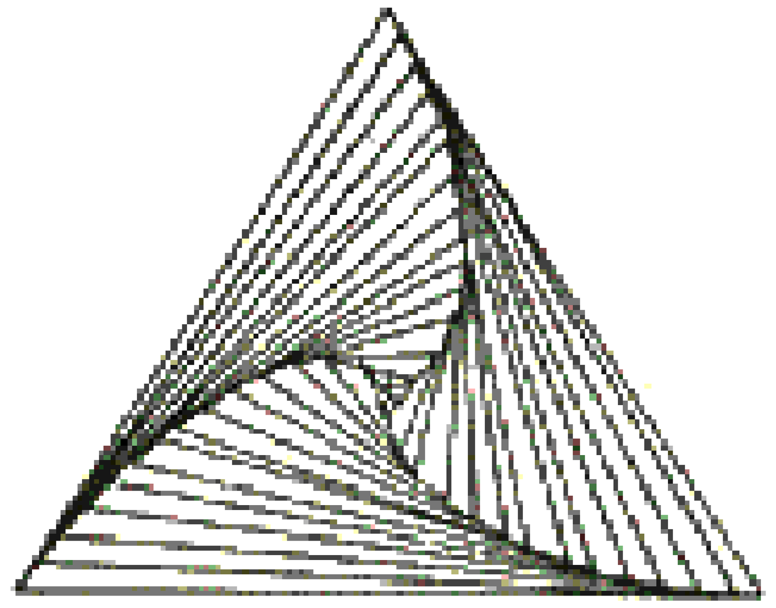


Fig. 2

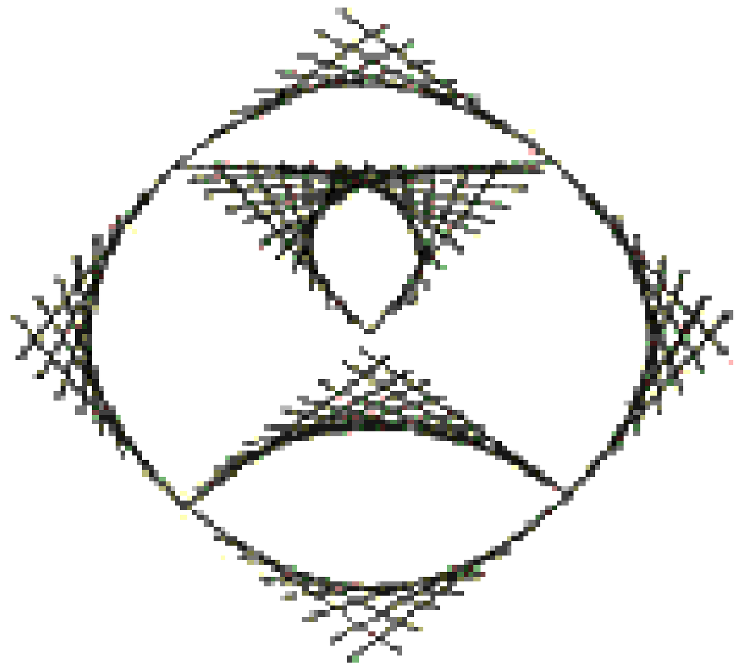


Fig. 3

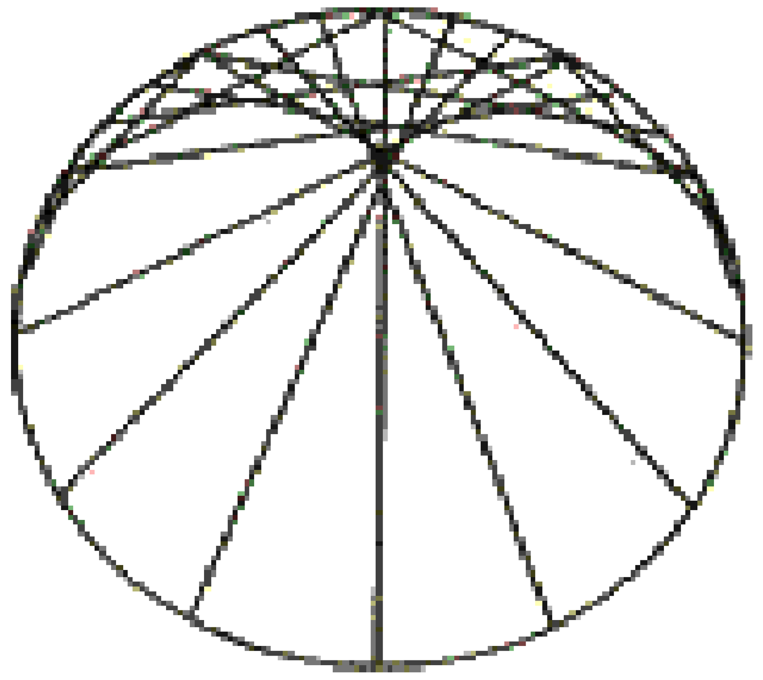


Fig. 4

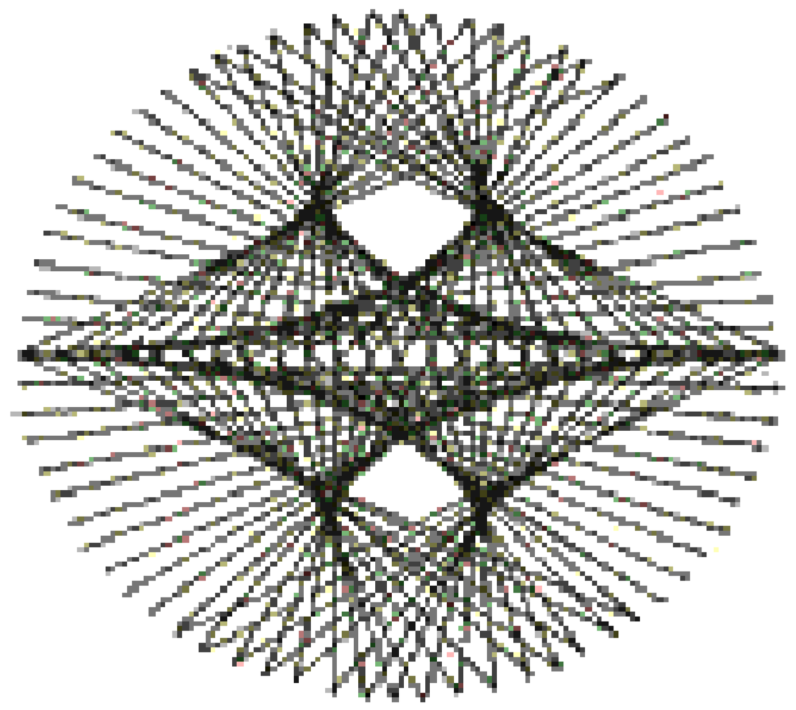


Fig. 5

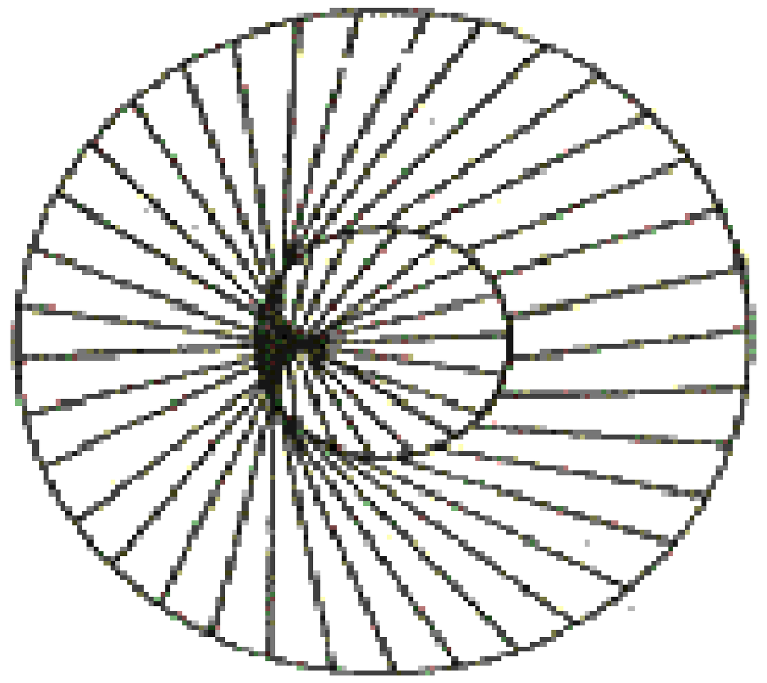


Fig. 6

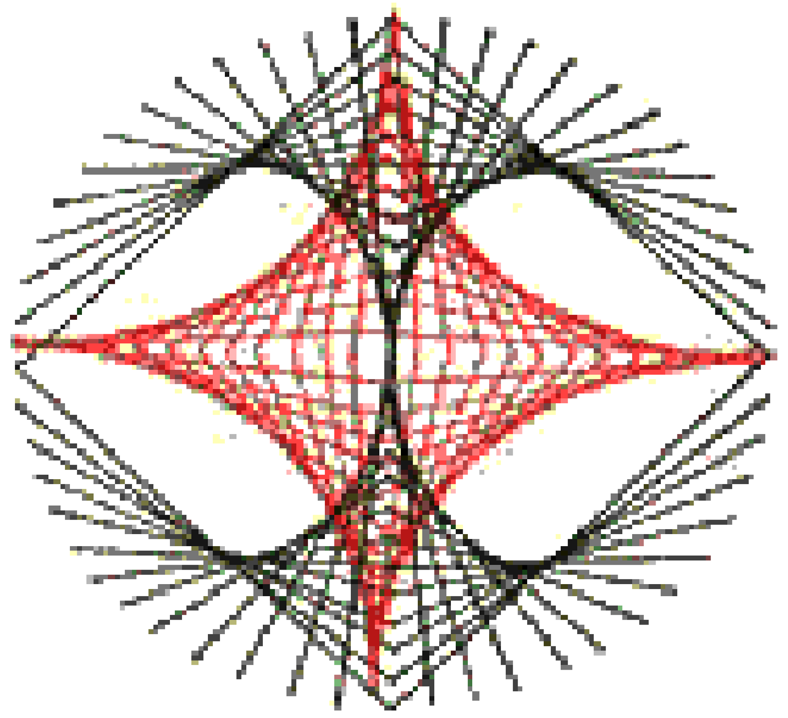


Fig. 7

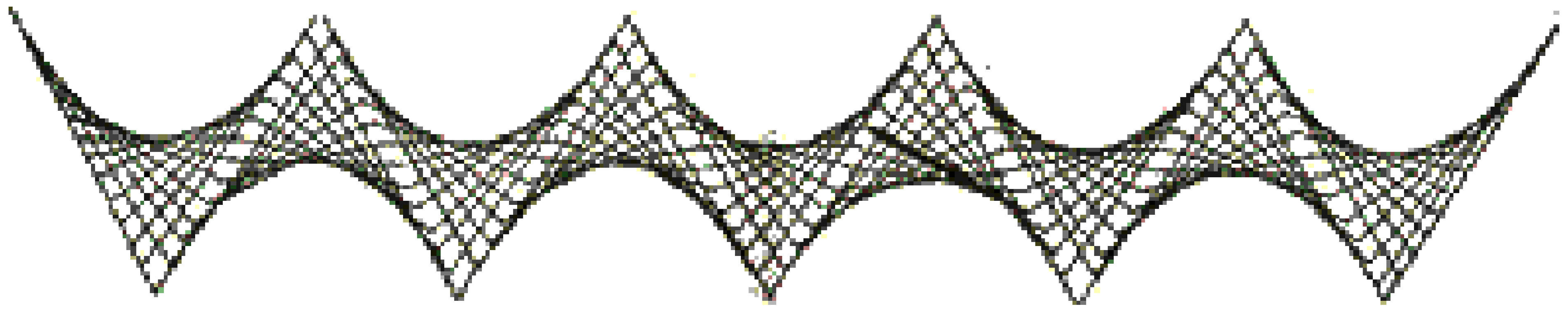


Fig. 8

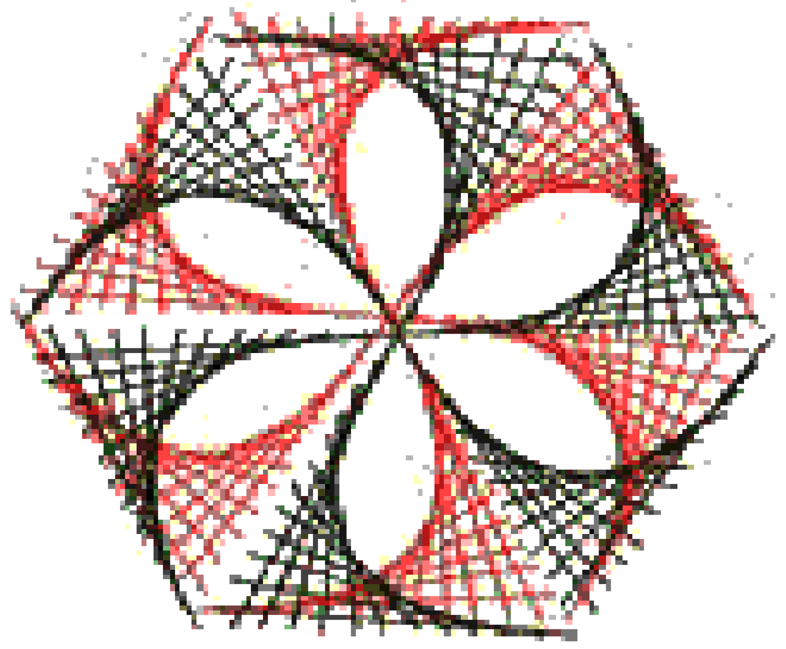


Fig. 9

14932 40529 76347 57481 19356 70911 01377 51721

00803 15590 24853 69066 92037 67192 20532 29094