QUADRATIC EQUATION SOLVER :
A 'Slide Bottle' which revolutionises the solving of quadratic equations :

Materials required : a plastic cylindrical bottle (several washing-up liquid brands are most suitable) ; a sheet of transparent plastic or thick cellophane paper ; two sheets of graph paper.

Construction :

1. Cut a sheet of graph paper to completely cover the curved surface of the bottle.

2. Draw on the horizontal and vertical axes for x and y, marking in values. A satisfactory range can be obtained by letting $-8 \leqslant x \leqslant +8$ and $-16 \leqslant y \leqslant +16$. (The range of x will limit the value of 'x' which can be found in $ax^2 + bx + c = 0$, while the range of y will limit the value of 'c').

Fig. 1

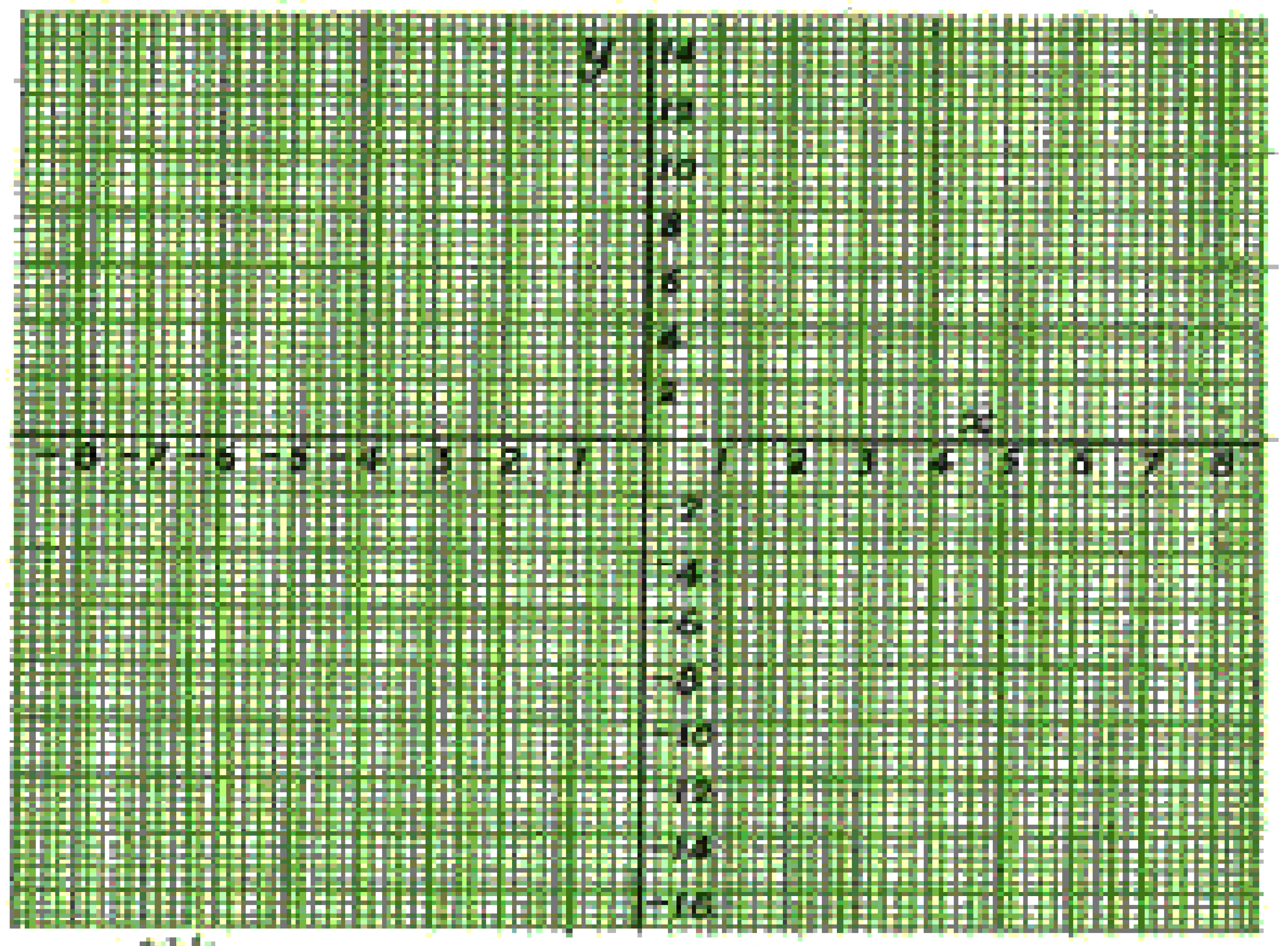


Fig. 3

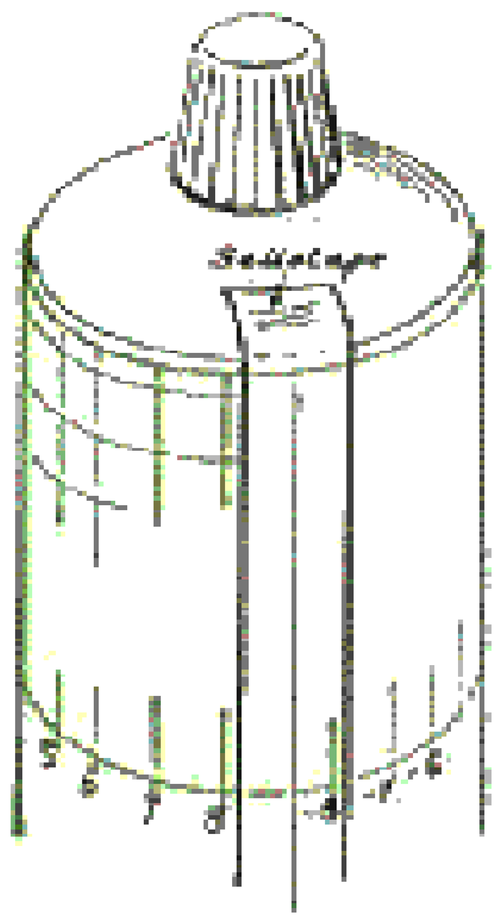


Fig. 2

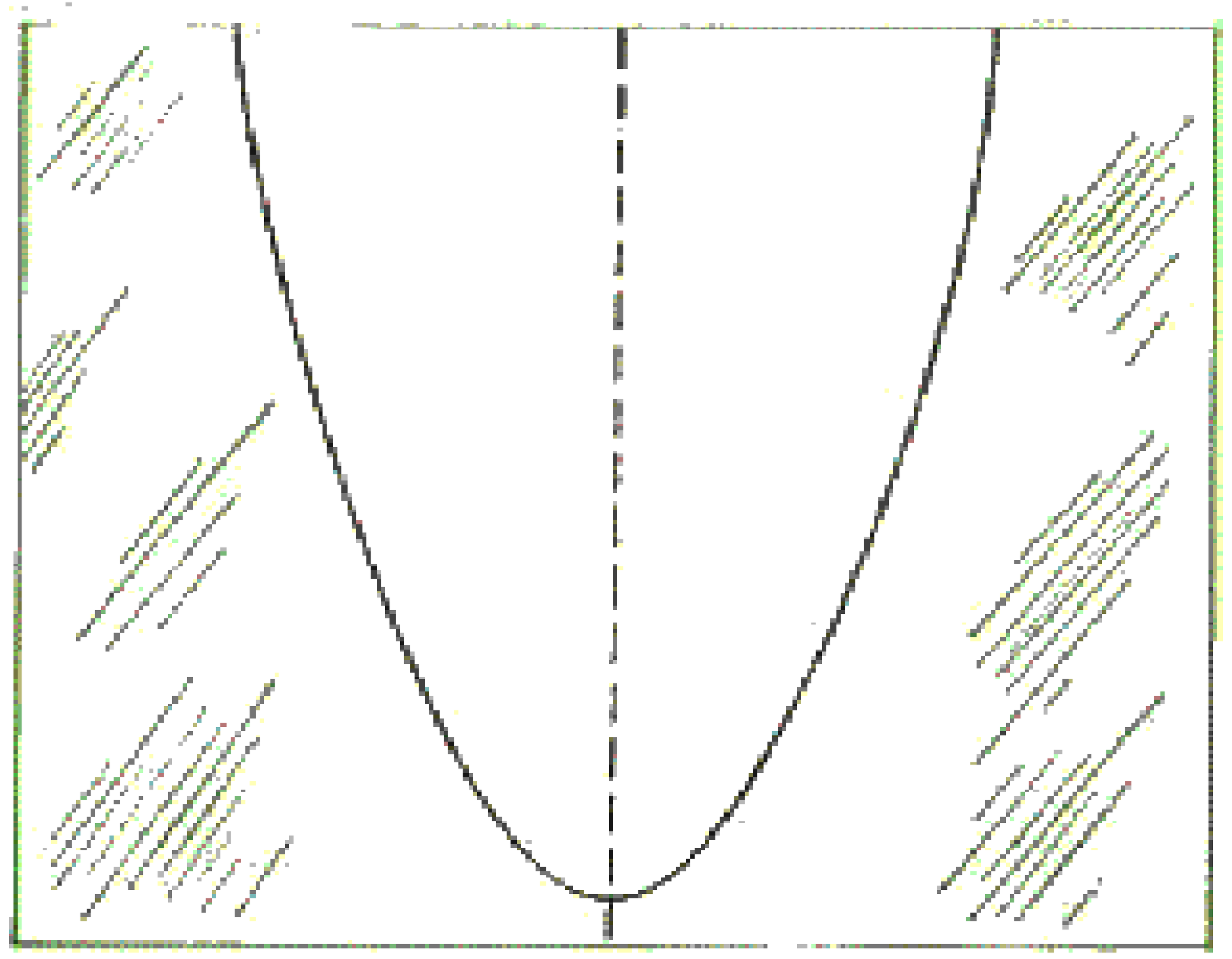
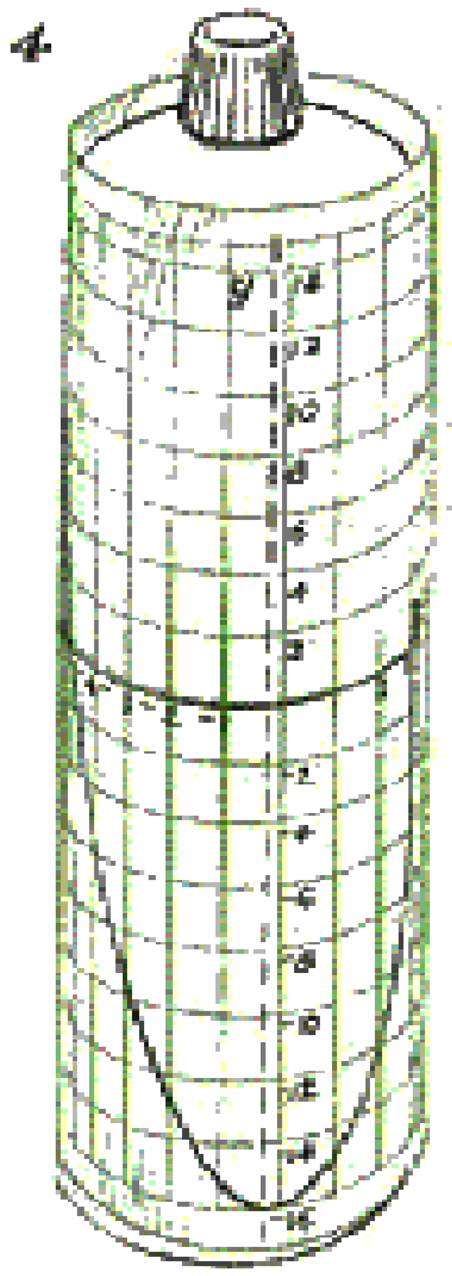


Fig. 4



3. Wrap the graph paper around the bottle securing with adhesive tape.

4. On the second sheet of graph paper mark in identical axes and, very carefully, draw the graph $y = x^2$.

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mathematical pie

No. 47 — *Editorial Address :* 100, Burman Rd., Shirley, Solihull, Warwicks, England — FEBRUARY, 1966

PASCAL'S TRIANGLE

The diagram shows a Chinese version of the triangle of Pascal, which was known to the Chinese in 1303 and rediscovered by Pascal some 300 years later.

Each line in the diagram corresponds to the coefficients of a binomial expansion, $(1+x)^n$, in ascending powers of x, the values of n varying from 0 to 8.

Use the diagram to write out the Chinese equivalent of the Arabic symbols for 1, 2, 3, 4, . . .

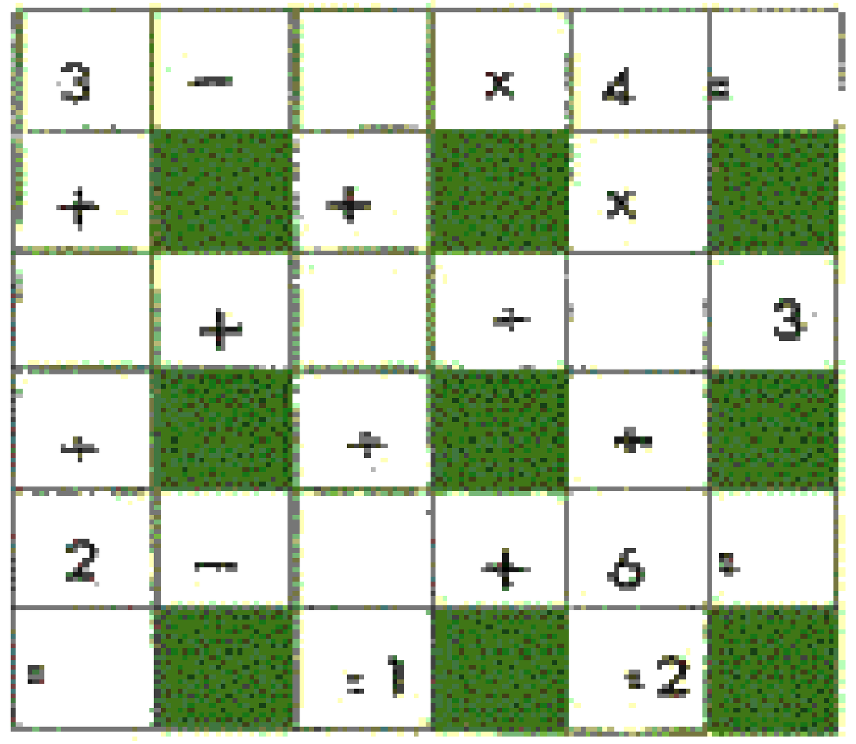
B.A.

(2,25) is a pair of numbers such that the difference between the product and the sum of the pair is 23, *i.e.*, $2.25 - (2+25) = 23$. Find three more pairs of numbers which have the same property. (Hint—the numbers you use are less than 14).

Now find a pair of numbers such that their product-sum difference is 215.

R.H.C.

03567 39177 75096 82887 47826 56995 99574 49066



WITHOUT A WORD

Each empty square requires one figure so that the working from top to bottom and from left to right is correct. B.A.

If a cubic foot of water weighs $62\frac{1}{2}$ lb., what is the approximate weight of water falling on an acre of land for a rainfall of 1 inch?

What is the weight of water falling on a city of area 50,000 acres? J.F.H.

A snail crawls up a post which is 20 ft. high. During the day it climbs 3 feet, but slips back 1 foot during the night. How many days will it take to reach the top? S.T.P.

EQUALLY DISPERSED

My "Gardeners' Handbook" says that onions should be planted 1 foot apart. How many could I plant inside and on the perimeter of a circle 6 feet in diameter if each is planted exactly 1 foot from its immediate neighbours? J.G.

A CUTTING PROBLEM

A wooden hemisphere is reduced to the largest possible cube by 5 plane cuts. If the hemisphere has a radius of 12 inches, what is the volume of the cube? J.G.

continued from page 372

5. Place the transparent sheet over this curve and trace the impression of the curve and the vertical axis of symmetry in Indian ink.

6. When dry, wrap this sheet around the bottle superimposing the axis of symmetry exactly over the y axis on the graph paper, with the turning-point as a minimum. Secure with adhesive tape to form a sliding sleeve which can be moved horizontally and vertically.

Use: to solve $x^2 - x - 6 = 0$.

1. Find minus half the coefficient of 'x', *i.e.*, $-\frac{1}{2}(-1) = +\frac{1}{2}$, and move the sleeve so that the axis of symmetry is in this position, *i.e.*, $x = \frac{1}{2}$.

2. Taking care to move the sleeve only vertically, arrange it so that the curve intersects the 'y' axis at the value of 'c,' *i.e.*, -6.

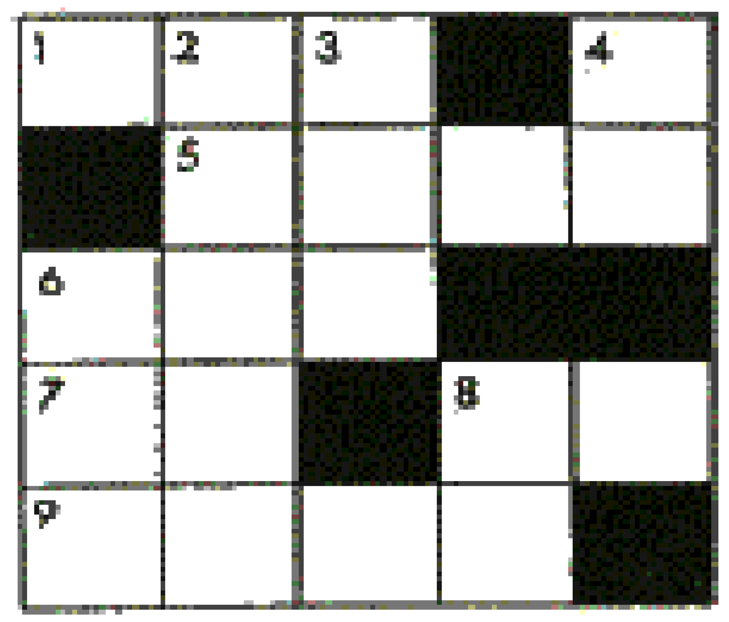
3. The roots of the equation can be found where the curve intersects the 'x' axis, *i.e.*, $x = -2$ and $x = +3$.

For equations of the general type $ax^2 + bx + c = 0$, first divide through by 'a' and then continue as in 1 — 3 above. D.I.B.

JUNIOR CROSS FIGURE No. 40

Submitted by Terence Morgan, Filton High School, near Bristol.

Ignore decimal points and work to the appropriate number of significant figures.



CLUES ACROSS:

1. Volume of a cylinder, radius 7″, height $2\frac{1}{2}$″, take $\pi = \frac{22}{7}$.
5. Sine of 22°36′.
6. Find the principal, in £, which yields £15 at 5% in 2 years.
7. The radius of a circle of area 24·62 sq. in., log π = ·4971.
8. Area, in sq. units, of a triangle with $a = 7$, $b = 20$, $\angle C = 30$.
9. Length of the diagonal of a square with sides 9″.

CLUES DOWN:

2. Square of 8·578.
3. Number of gallons in 60 bushels.
4. $n = 13$, $x = 3$, and $y = 4$. Evaluate $\dfrac{nx + ny}{x + 4}$.
6. The square of 11.
8. The number of yards in $1\frac{1}{2}$ chains.

SOLUTIONS TO PROBLEMS IN ISSUE No. 46

FOR EXPERIENCED MATHEMATICIANS

Owing to the large number of solutions, the answer will be given in issue No. 48.

STEPPING IT UP!

(*a*) $x = 2$, (*b*) $x = 2$, (*c*) $x = 2$, (*d*) $x = \log 6/\log 3$, (*e*) $y = 4$, (*f*) $y = \log 10/\log 2$, (*g*) $y = \log 11/\log 4$.

PASSING BY

Each train is [illegible] mile, or a little over 162 yards.

SENIOR CROSS FIGURE No. 44

CLUES ACROSS: (1) 355, (4) 426, (7) 33, (8) 21, (10) 231.
CLUES DOWN: (1) 35, (2) 54, (3) 36, (5) 242, (6) 133, (7) 32, (9) 12.

WITHOUT A WORD

$6 + 4 - 3 = 7$, $1 + 3 + 2 = 2$, $3 \times 2 \div 6 = 1$.

A STRIKING PROBLEM

Each pause takes six-fifths of a second; eleven strokes takes ten pauses, hence twelve seconds. Twelve takes thirteen and one-fifth seconds.

A BREAKFAST-TIME PROBLEM

Toast side 1 of slices 1 and 2. Turn slice 1 and replace slice 2 by slice 3 side 1. Turn slice 3 and replace slice 1 by side 2 of slice 2.

BULL'S EYE

The radius of the outer circle is $\sqrt{10}$ inches.

UPWARDS EVER UPWARDS

The least journey is twice the height of the cube to the edge of the top plus half to the centre.

JUNIOR CROSS FIGURE No. 39

CLUES ACROSS: (1) 361, (4) 29, (5) 28, (6) 987, (8) 697, (10) 101.
CLUES DOWN: (1) 32, (2) 6999, (3) 18, (5) 27, (7) 871, (8) 62, (9) 1.

ENCIRCLING MOVEMENT

The fourth side is $\frac{1}{2}(5 + \sqrt{73})$ or 6.77 inches.

CLOCK ARITHMETIC No. 5

$a^{2n+1} = a$ B.A

intersect at V. X is distant d_1 from V as before. Using X as pole, draw a series of rays intersecting AA^1 and BB^1. Choosing one of these lines, say AA^1, mark off distances $= d_2$ along the rays on both sides of the line giving a series of points Y_1 Y_2 - - - and Z_1 Z_2 - - -. By joining the two sets of points as shown, two parts of a curve (called a *conchoid*) are drawn, one on either side of AA^1. If the curve cuts BB^1 at P_2 and Q_1, the rays XP_2Q_2 and XP_1Q_1 provide solutions of the problem. Note that a second conchoid can be drawn by marking off d_2 from BB^1. This conchoid, shown in broken lines, provides the same solutions as the first.

For this method of solution, which can be worked by anyone capable of using a ruler and dividers, nothing has to be known about higher mathematics. It is a useful exercise to construct a set of conchoids by keeping XR (Fig. 3) constant and taking different value for RS $(=d_2)$: when RS is made greater than XR, the part of the curve on the same side of AA^1 as X assumes an interesting form.

Which method of solution appeals to you as (*a*) more elegant, (*b*) more practical, (*c*) having taught you most? J.F.H.

The operation * is defined by

$$a*b = \frac{a+b}{1+ab}.$$

Show that the operation is associative and evaluate

(i) $1*2*3*\dots\dots*n$,

(ii) $\frac{1}{2}*\frac{1}{4}*\frac{1}{6}*\dots\dots*\frac{1}{2n}$. C.V.G.

AL'S ANALYSIS

Do you remember our American friend Al Gebra, from issue No. 43, and his problem about the number of ways of going to work? Do you also remember how Pascal's triangle gave him the solution?

Al has a son, Little Alph, and last night Al found him struggling with his Mathematics Assignment (homework to you). Alph was groaning over the fact that old Chalky had given him $(a+b)^7$ to work out and it was going to take him hours, and he'd fixed to go to a movie with his friend Trigger......Al had a look at what his son had written and it struck him as vaguely familiar. Here it is:

$$a+b$$
$$(a+b)^2 \qquad a^2+2ab+b^2$$
$$(a+b)^3 \qquad a^3+3a^2b+3ab^2+b^3$$

Then the penny dropped and seizing a pencil Al wrote it out again without the *a*'s and *b*'s.

1 1

1 2 1

1 3 3 1

and he was able to add the next line with very little work:

1 4 6 4 1

It was his old friend Pascal's Triangle again. Do you remember how to go on from line to line? If you do you will be able to write down the expression for $(a+b)^7$ as easily as Al did. Alph and Trigger said it was a good movie. Al's efforts will help you with the problem on the front page. R.M.S.

58382 92041 97484 42360 80071 93045 76189 32349

ODDS ON

Everything was going with a swing at the Prefects' Valentine Dance. In the Paul Jones, David was pleased to find that his first partner was Jacqueline. He could not believe his luck when they were partners again for the second time and when the circles stopped for the third time, he was staggered to find himself face to face with Jacky once more.

" It's a chance in a million," he said to her (amongst other things).

" Don't be silly, Dave," said she, " it's only about 1 in 6,000 and in any case I'm the lucky one because there are three more girls here than boys."

The rest of the conversation was rather private so we'll skip it. Actually, Jacky was wrong because she had not noticed that Rob and Lindsey were sitting out in the corner behind the piano and this reduced the odds to 4912 to 1 against. How many prefects of each sex were at the party? R.M.S.


Pythagoras and his Squares!


Archimedes and the Twist!

If a bag contained twelve balls, four red, four white and four blue, what is the smallest number you would have to withdraw to ensure that you had

(i) Two balls of the same colour

(ii) One ball of each colour

(iii) Three red balls? S.T.P.

SENIOR CROSS FIGURE No. 45

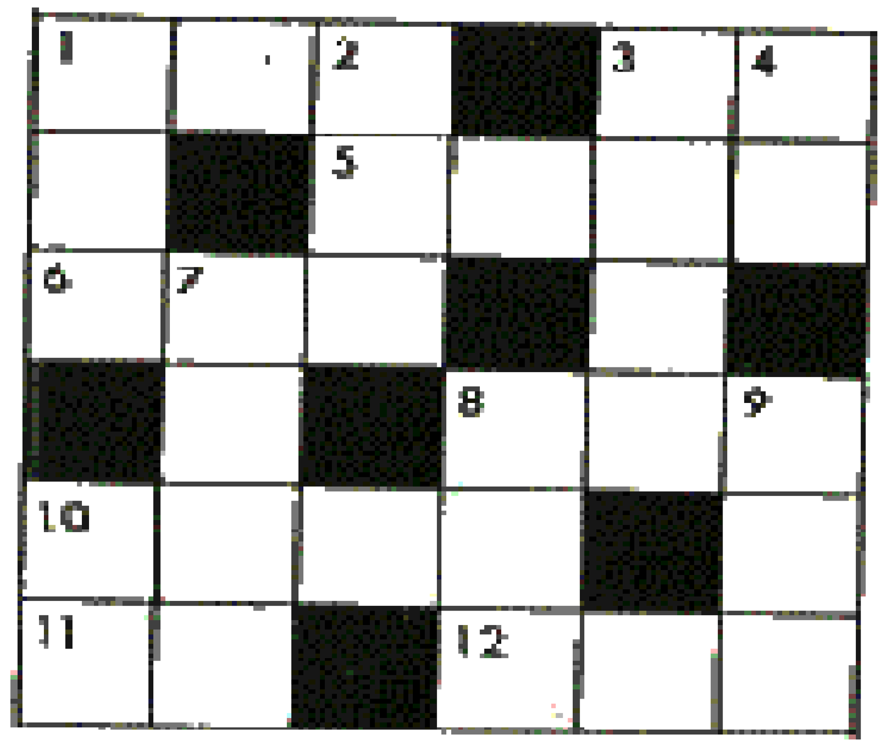
Submitted by Paul J. Castle, King Edward's School, Birmingham.

CLUES ACROSS:

1. $2y^2 + y$.
3. a.
5. $(b+2)c$ (palindromic).
6. $xz + \frac{1}{2}y$.
8. $d(x-y)^2$.
10. $\left(\frac{y^2}{2}\right)^2$
11. $2x + z + 2(d-x)^2$.
12. $2y^2 + z + 2$.

CLUES DOWN:

1. $y + 3z$.
2. bz.
3. cx.
4. $3a$.
7. fx.
8. $(z-1)(x-2)$.
9. y^3.
10. $z - 2y$.



Only whole, positive numbers are involved in the working and the solutions. No guess work is necessary. The values of a, b, c, d, e, f, x, y, and z can be derived from the clues.

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Mathematical Elegance

Fig. 1

Fig. 2 (Continued)

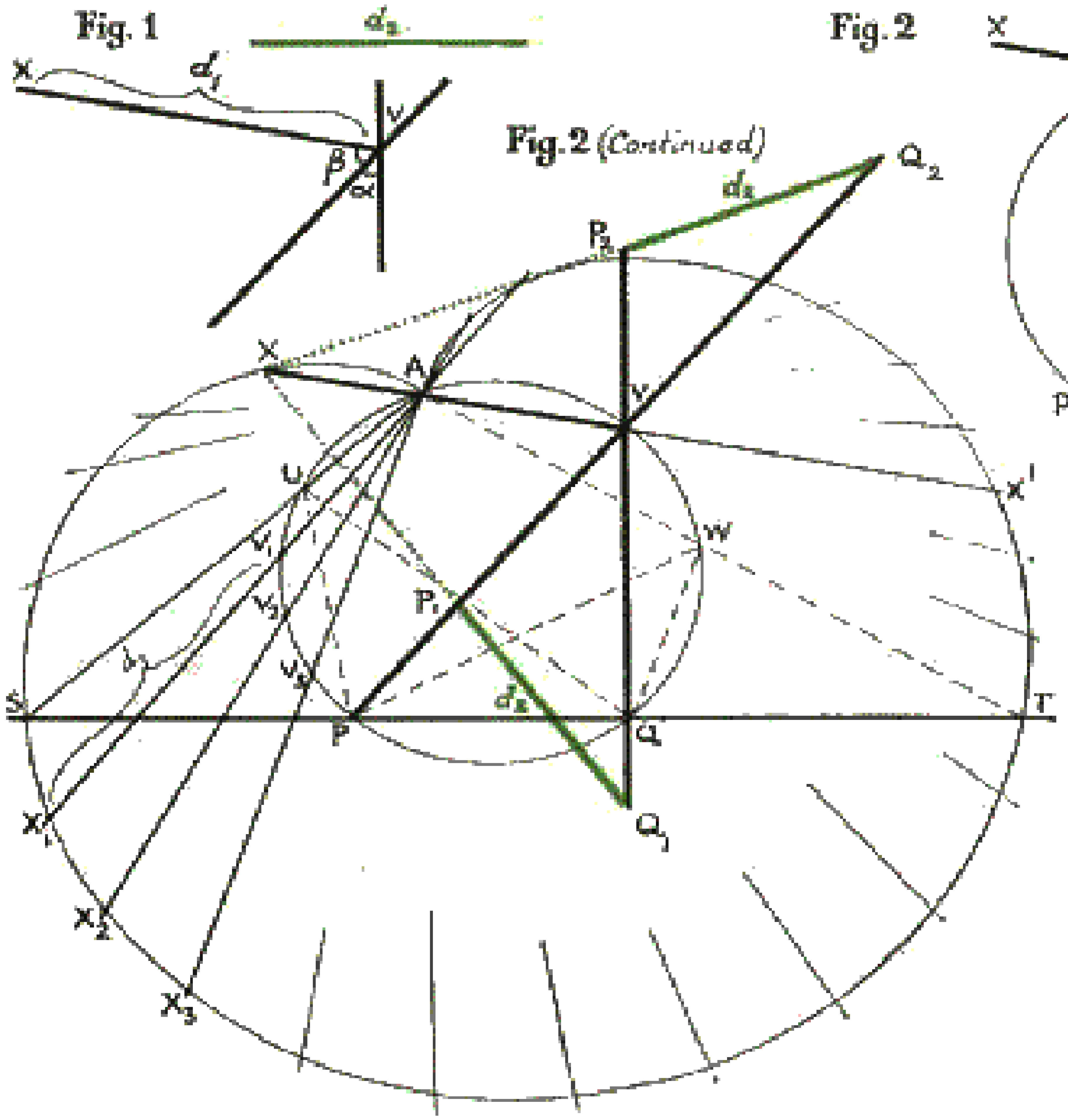


Fig. 2

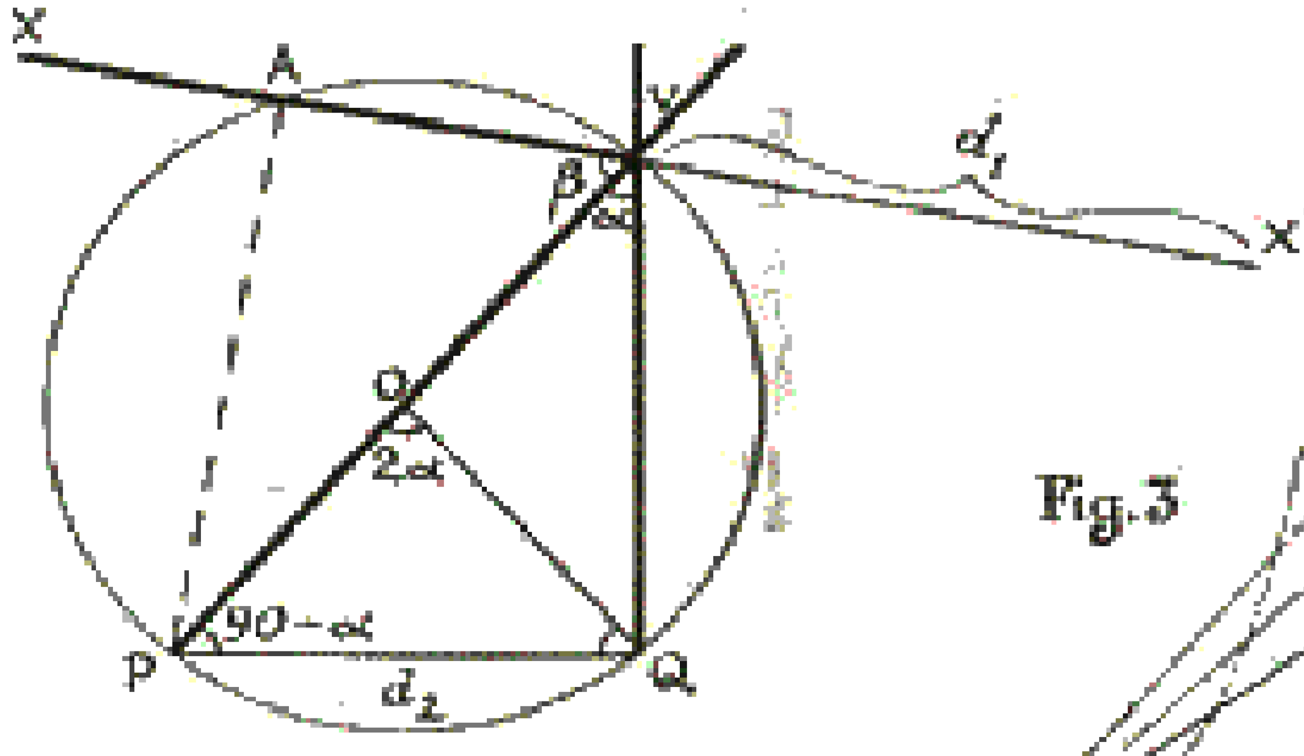
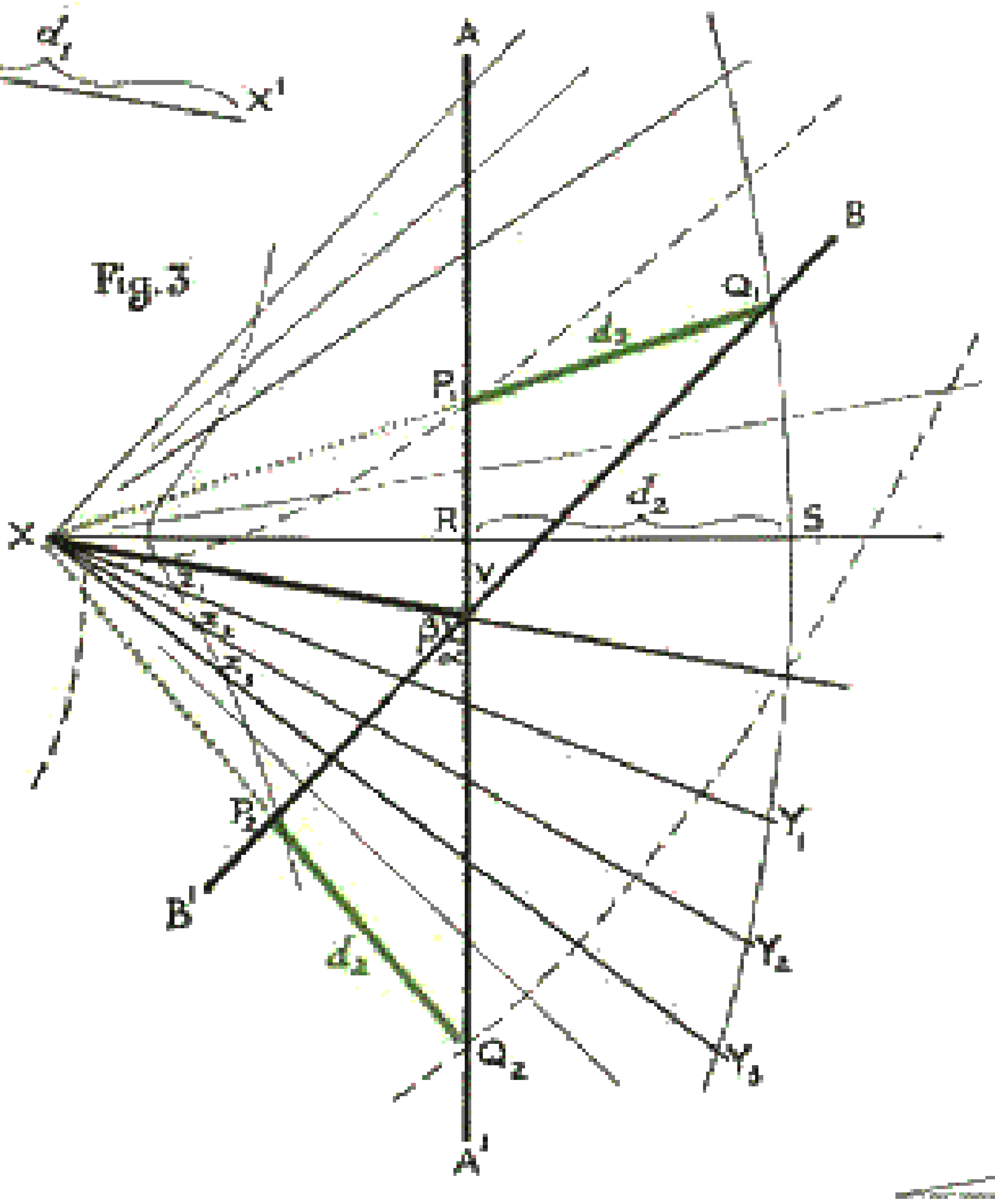


Fig. 3



One of the interesting features of mathematics is the fact that, if a problem *is* soluble, there are usually several ways of arriving at a solution — some of these ways may be more direct than others. As you become more proficient in your mathematics, you will learn to appreciate " elegance " in working for a solution.

An elegant solution may be described as one that is reached in the fewest possible steps — each step being clear and logical.

When you are learning mathematics, however, the solving of problems serves a useful purpose in gaining familiarity with a useful theorem or method; a longer way round, therefore, may sometimes provide a better exercise.

As an example, take the problem illustrated in Fig. 1. Two straight lines intersect at V making an angle α. A point X is distant d_1 from V and XV makes an angle β with one of the lines. A line is to be drawn through X, intersecting the two lines in such a way that they cut off a length equal to d_2.

Method 1.—making use of circular segments and subtended angles. Draw a baseline and cut off a distance PQ equal to d_2 (Fig. 2). At P and Q draw lines making an angle $(90^0-\alpha)$ with PQ so that they meet at a point O: then angle $POQ=2\alpha$. With centre O and radius OP $(=OQ)$ describe a circle passing through P and Q. PQ subtends an angle equal to α at any part of the segment on the same side of PQ as O.

Take any point V at random on this segment and join VP, VQ. Through V draw a line making an angle β with VP and mark X and X^1 on it—distant d_1 from V, one on either side. If VX cuts the circle at A, then AP will subtend an angle β in the segment AVP. If the point V is moved round the circumference so that XVX^1 always passes through A, the points X and X^1 trace a curve or locus ; this is the heart-shaped curve $AXSTX^1$. An easy way is to mark a series of points V_1 V_2 V_3 - - - on the circle and to draw lines through A and these points. With a pair of dividers mark off the distance d_1 on each to give points such as X_1 X_2 X_3 - - -. Join the points.

The curve $AXSTX^1$ is called a *limacon*; the point A is called the *pole*. The curve cuts PQ produced at S and T. Since US is equal to d_1 and the angle PUQ is equal to α, the configuration $SPQU$ is one solution of the problem ; similarly, $TWPQ$ is another. Mark off $VP_1=UP$ and $VQ_1=UQ$ also $VP_2=WQ$ and $VQ_2=WP$. The lines XP_1Q_1 and XP_2Q_2 provide the two solutions.

Method 2.—much shorter ; more elegant? (Fig. 3). Let AA^1 and BB^1

Continued on next page