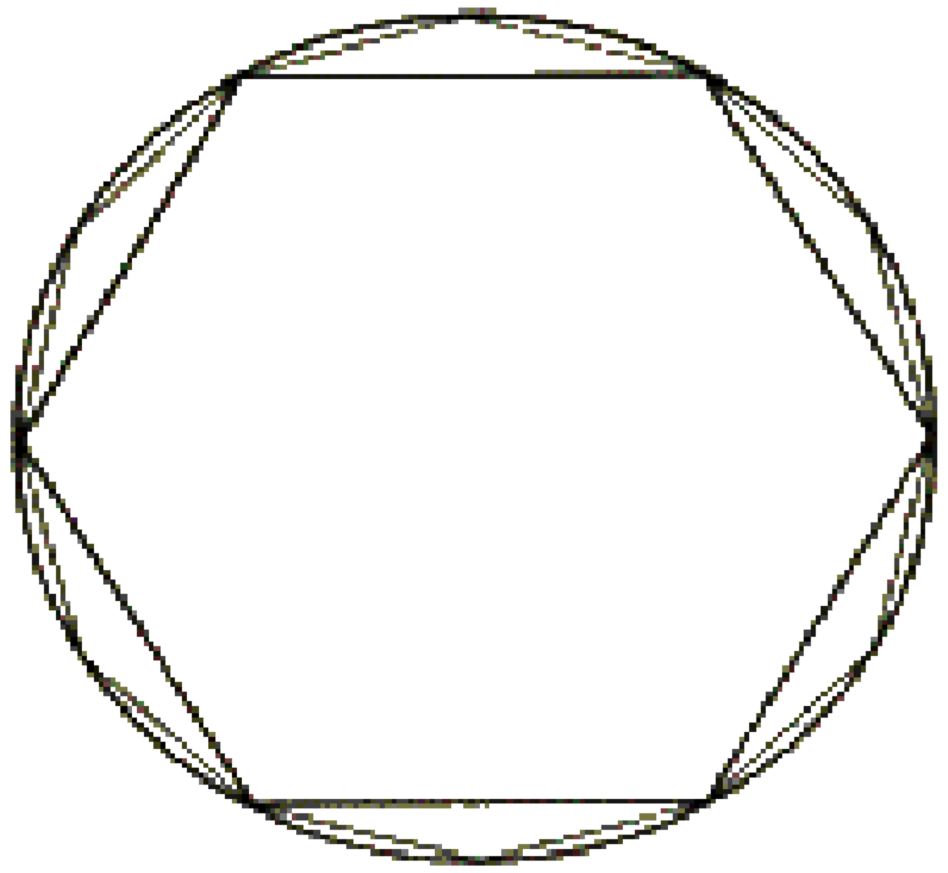
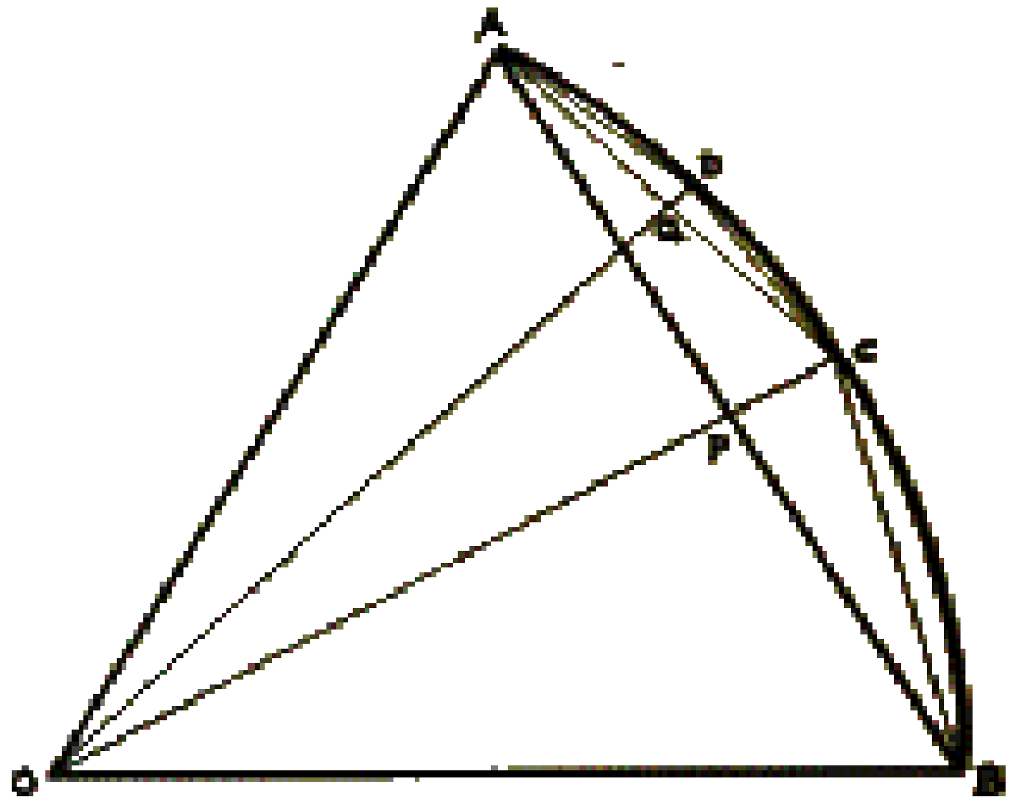
EXHAUSTION

The crude idea of the measurement of the area of a figure was to count the number of unit squares which would cover it. It was a great advance when it was shown that the area of a rectangle is found by multiplying the length by the breadth, even if these are fractions. As soon as it is accepted that a square inch need not be a square, it is fairly easy to calculate the areas of parallelograms and triangles, or any figures with straight sides. To find the areas of figures with curved sides is more difficult. In the third and fourth centuries B.C., the Greek Mathematicians invented a method which was still in use in the seventeenth century A.D. Rather surprisingly, they used the theorem of Pythagoras to find the area of a circle. Anyone who can calculate a square root can calculate π accurately to several decimal places. (The Greeks had a harder task as they had to work out their square roots as fractions).





We can start by fitting a regular hexagon inside a circle of unit radius. In the figure ABO is an equilateral triangle and P is the mid-point of AB. Therefore AP is 0.5 inches.

Then $OP^2 = 1^2 - 0.5^2 = 0.75$ and $OP = 0.86603$.

The area of the hexagon is therefore $6\times\frac{1}{2}\times1\times0.86603 = 2.59809$.

In the spaces between the hexagon and the circle we fit six isosceles triangles, like triangle ABC. $CP = 1 - 0.86603 = 0.13397$.

So the area of the six triangles is $6\times\frac{1}{2}\times1\times0.13397 = 0.40191$. Into the twelve places that are left we fit twelve more isosceles triangles like ACD.
$AC^2 = AP^2+CP^2 = 0.5^2 + 0.13397^2 = 0.26794$, and $AC = 0.51763$
$OQ^2 = OA^2 - AQ^2 = 1^2 - 0.258815^2 = 1 - 0.06698 = 0.93302$
$OQ = 0.96592$ and $DQ = 0.03408$ So the area of the twelve triangles is $12\times\frac{1}{2}\times0.51763\times0.03408 = 0.10584$. The method consists of fitting more and more isosceles triangles into the gaps until all the space is exhausted. Unfortunately, the mathematician is always exhausted first. If we stop now and add together the areas of the hexagon, the first six triangles and the next twelve triangles, we find the area of the 24 sided figure is 3.1058. By making two more steps Archimedes showed that the area of the 96-sided figure inside the circle was more than $3\frac{10}{71}$ and that the area of the 96-sided figure outside the circle was less than $3\frac{10}{70}$ or $3\frac{1}{7}$, and so showed that the area of the circle was between these two values. By A.D. 300, the value of π was known

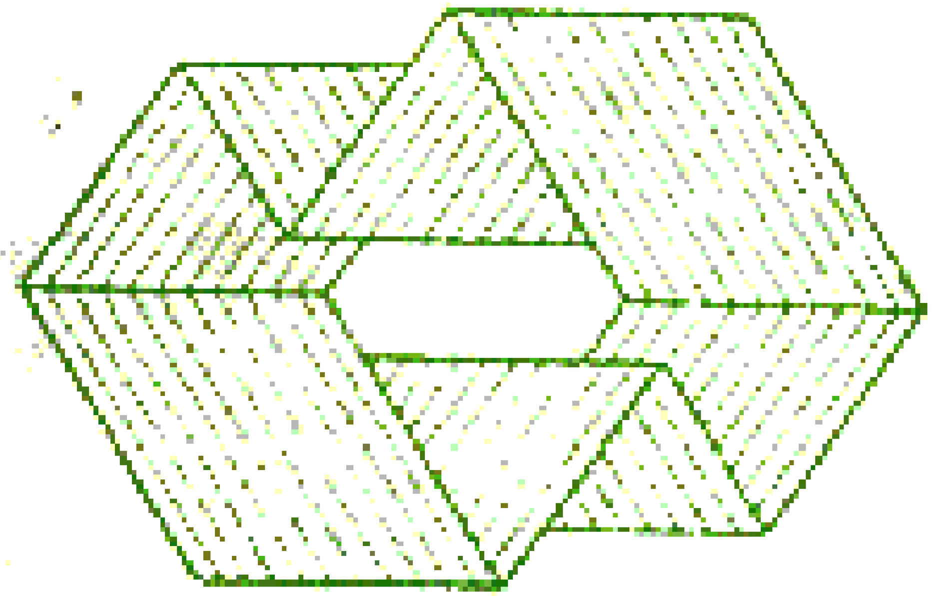
Continued on page 395

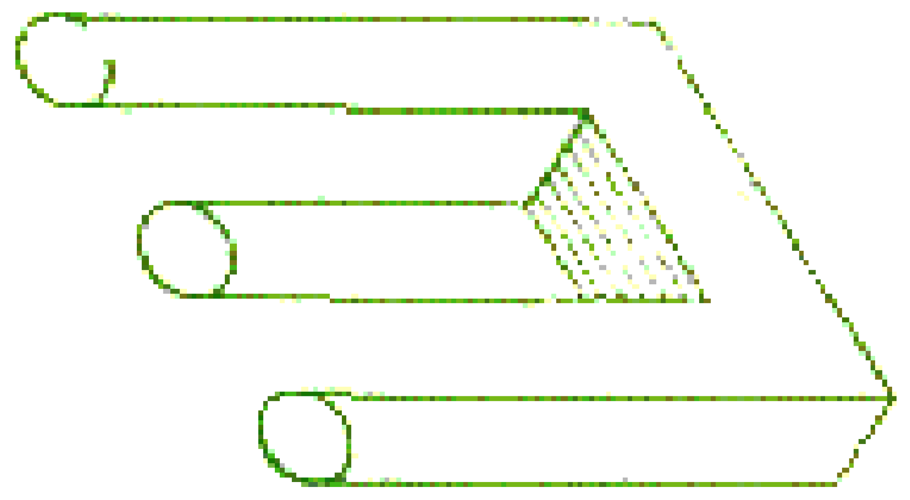
27056 23526 60127 64848 30840 76118 30130 52793

mathematical pie

No. 50 — *Editorial Address:* 100, Burman Rd., Shirley, Solihull, Warwicks, England — FEBRUARY, 1967

OPTICAL ILLUSIONS

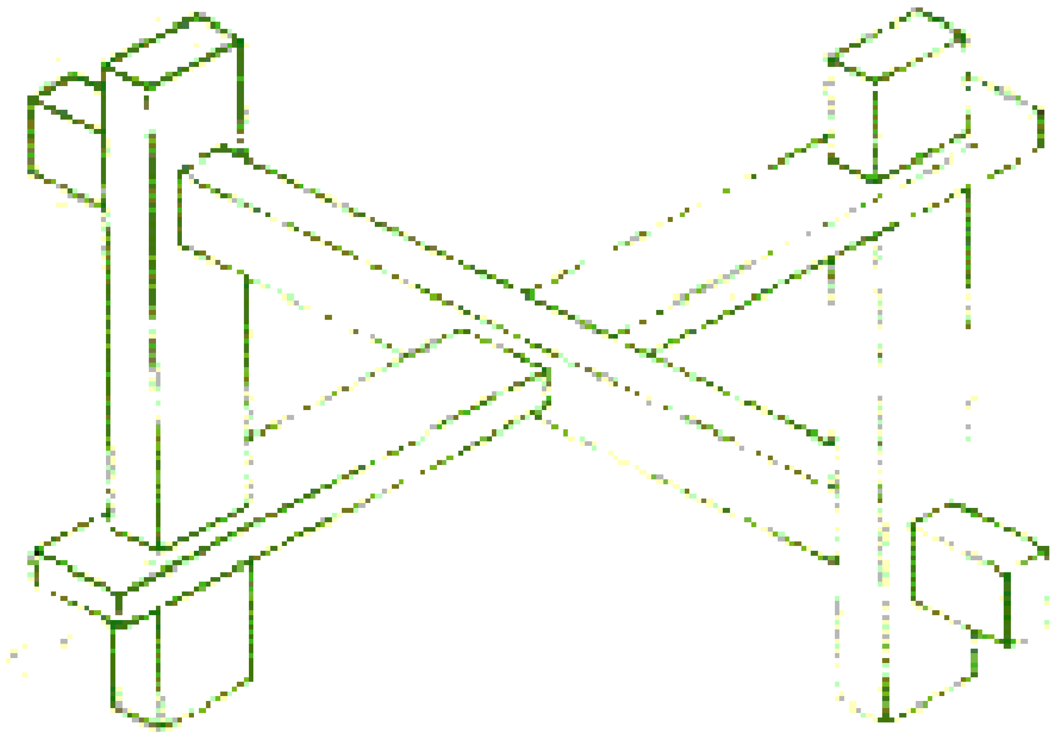


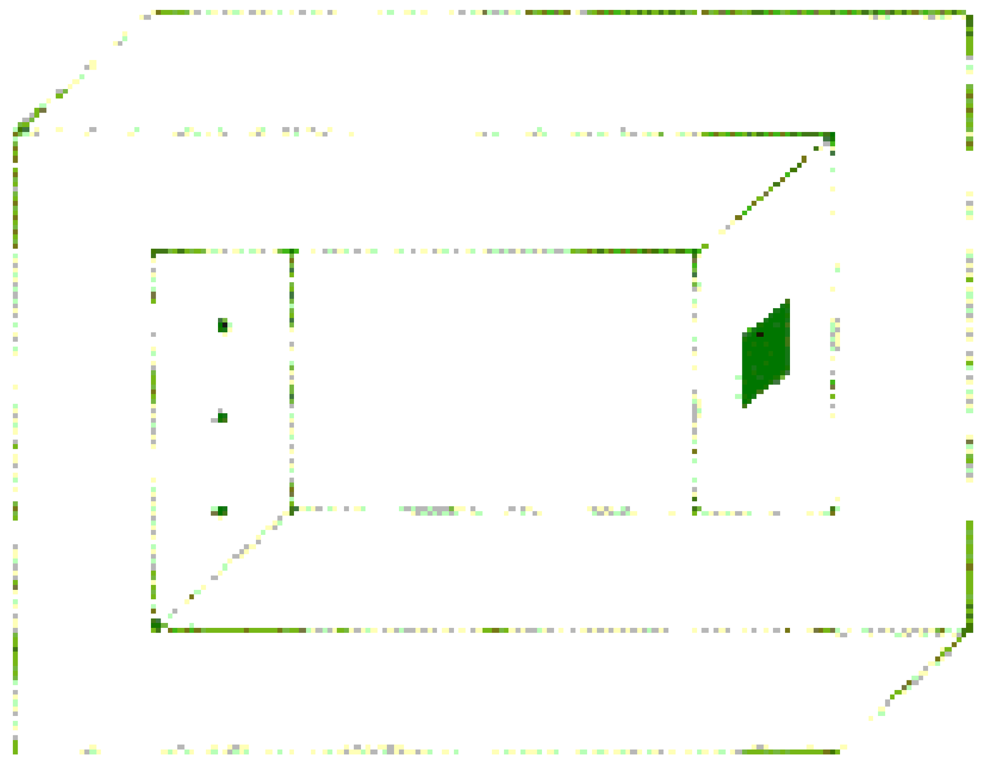


The brain interprets what the eyes see in the light of past experience, taking into account the apparent size, shape and distance of the object from the observer. Hence, the effectiveness of an illusion depends upon the experience of the observer. Serious illusions may occur when there is no framework against which an object can be judged.

The figures show four different illusions in which the brain interprets the picture in one way when concentrating on one part and in another when the concentration is moved to a different part of the figure. It is possible to fit the upper right figure into the lower right figure if the scales are changed, so that the three circular rods fit on to the three dots and the other end fits on to the green patch. Nuts to fit these rods have been designed in various drawing offices, but of course they have not become a commercial proposition.

It is hoped that more care will be exercised when interpreting geometrical figures. B.A.





25437 09069 79396 12257 14298 94671 54357 84687

MATHEMAGIC

All that you need for this Party piece is one month page from an old calendar. Ask a friend to put a square round any 4×4 block of dates. At this stage you write down a number N on a piece of paper, fold it over so that it is concealed and give it to someone to hold.

Now ask your friend to circle one of the 16 numbers and then strike out the rest of the row and column containing it. Ask another friend to circle one of the remaining 9 numbers and strike out the rest of the row and column leaving 4 numbers. Repeat with one of the remaining four numbers, striking out the rest of the row and column leaving one number which you circle.

Ask your friends to add the four circled numbers and compare their result with the number N — lo and behold they are the same!

In the solutions you will find how to construct the number N — but before you turn to it, can you think out how to do it? R.M.S.

CUBISM

Each face of a cube is painted with one of six different colours, all six colours being used on each cube. How many distinguishable cubes can be produced in this way? S.T.P.

7	+		−		= 8
+		+		+	
	+		−		= 6
−		−		−	
	−		+		= 7
= 8		= 9		= 2	

WITHOUT A WORD

Each empty square requires one figure so that the working from top to bottom and from left to right is correct.

D.I.B.

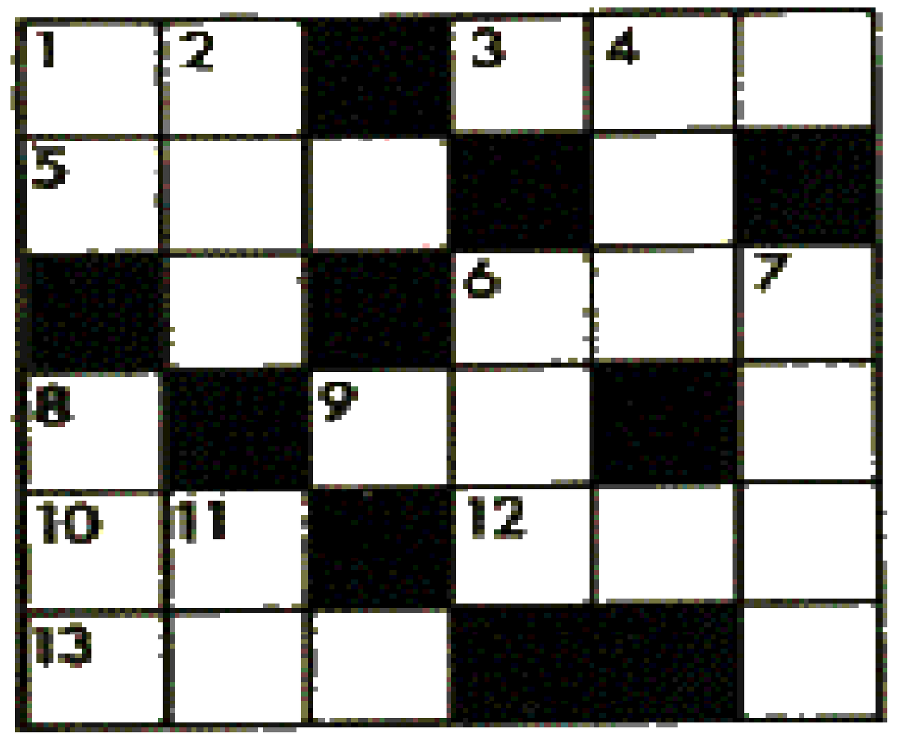
AS EASY AS A B C

ABC is a triangle in which the bisectors of the angles at B and C meet the opposite sides at D and E respectively. BD is equal to CE. Can you show that the triangle ABC is isosceles? J.F.H.

STAMP COLLECTORS' CORNER No. 23

Pierre Simon, Marquis de Laplace, 1749-1827, who was the son of a peasant farmer became a Professor in the Royal Military School in Paris. He was made a Count by Napoleon and a Marquis by Louis XVIII. To his contemporaries his chief work seemed to be in Astronomy where he solved many problems on the satellites of the planets, but he also worked on probability and algebra where he developed much of the foundations of the modern theory of matrices. C.V.G.

JUNIOR CROSS FIGURE No. 42



Clues Across

1. a + b + d.
3. a^3.
5. $(a - b - d)^2$.
6. $(b + d)^2$.
9. $3(a + b)$.
10. Twice 1 across.
12. $a(a^2 - a)$.
13. $d^3 - 1$.

Clues Down

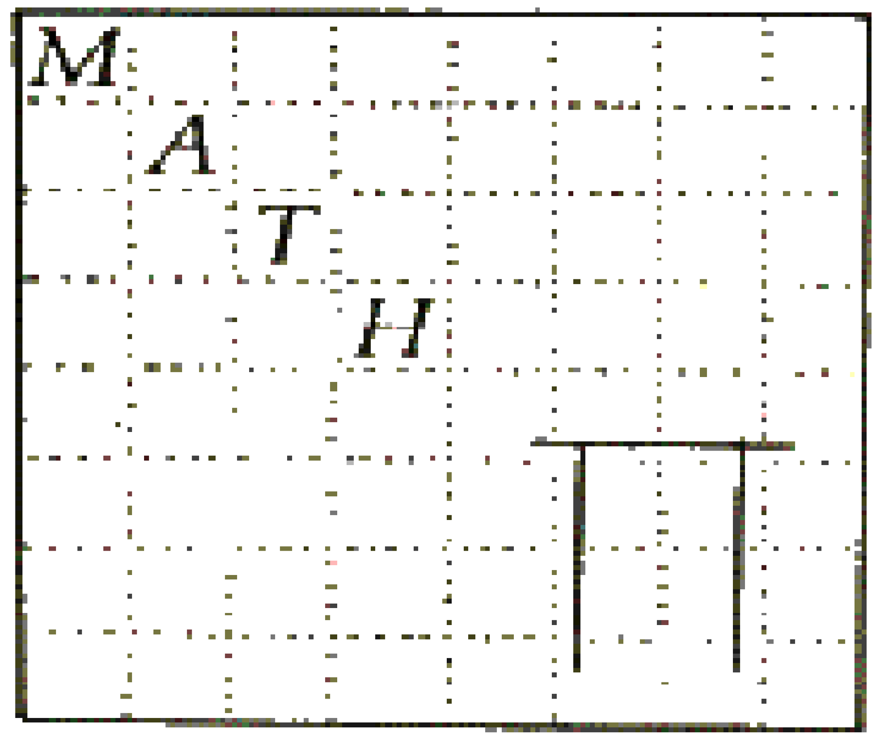
1. $2(a + b)$.
2. 4bd.
4. $2(a + b)^2$.
6. $9(2b + 3a)$.
7. $9d^3$.
8. $b^2 + d^2$.
11. b^2.

A.B.

$a = 5$, $b = 7$, $d = 10$.

CUTTING UP THE PIE

Cut the square into four congruent pieces such that each has one letter and a piece of PIE. J.F.H.



SOLUTIONS TO PROBLEMS IN ISSUE No. 49

We are still receiving letters on General Taichoff and the Scalene Triangle. These will be dealt with in the next issue.

SENIOR CROSS FIGURE No. 46

CLUES ACROSS: 1. 121; 3. 729; 5. 612; 7. 81; 9. 31; 10. 46656; 11. 94; 12. 18; 14. 256; 16. 125; 17. 441.
CLUES DOWN: 1. 148; 2. 16; 3. 72; 4. 961; 6. 15625; 8. 144; 9. 361; 11. 961; 13. 841; 14. 25; 15. 64.

JUNIOR CROSS FIGURE No. 41

Clue 2 down should have read 111 ÷ 1 reversed.
CLUES ACROSS: 1. 341; 4. 422; 11. 343; 12. 024; 14. 1011.
CLUES DOWN: 1. 30341; 2. 14301; 3. 32; 10. 224; 13. 21.

SOLUTIONS TO PROBLEMS IN ISSUE No. 50

Two minuses make a plus. Mr. Barker of Hymers Colleges points out that $(8888-888) \div 8 = 1{,}000$. This makes for any number from 1 to 9 as well as 8.

WITHOUT A WORD: 7+4—3=8. 4+6—4=6. 3—1+5=7.

CUBISM: There are 30 different cubes. MATHEMAGIC: N = 2 [1st + last dates].

INITIAL ALGEBRA 2

The only possible amounts I have found are 57s. 0d. + 14s. 6d., which represents purchase tax of 25%. This gives a code word C??TIAN?Y is presumably CERTIANLY.

FUN WITH NUMBERS No. 8: $10 = 9\frac{99}{99}$ among others.

THE ETERNAL TRIANGLE: The number of triangles is 118.

SEQUENCES

1. The sequence continues 12, 13 and the scale is five.
2. 100 101 111 and the scale is two.
3. 8, 9 @, *, 10, 11, 12, and the scale is twelve where @ means ten and * means eleven.

Another solution suggested by a student was 8, 9, A, B, and the scale is C. B.A.

Continued from page 396

correct to six decimal places. If you have the issues of *Mathematical Pie* from Number 18, you have the decimal equivalent of π to 9,270 places of decimals.

MORE MATHEMATICAL SHORTHAND — PART 1

In issue No. 46 we were introduced to $\int$, the sign of mathematical fiddling, and previously we have met ! These are both mathematical symbols to signify, as briefly as possible, some lengthy process. $\int$ for example, is a shorthand way of writing a particular kind of addition, and ! an important type of multiplication.

Now you are all familiar with the sort of riddle-me-re which starts

My first is in ladder and also in stairs
My second in apples and also in pears

Strange though it may seem, a mathematician may also need to solve problems like this, and he has invented a whole set of symbols to help him. For example, he might write

$$A = \{l,a,d,e,r\}$$

meaning that A is the set of different letters which appear in the word 'ladder.' Similarly he might write

$$B = \{s,t,a,i,r\} \qquad C = \{a,p,l,e,s\} \qquad D = \{p,e,a,r,s\}$$

for the letters used in writing 'stairs,' 'apples' and 'pears.' Then if he is solving the first part of the riddle-me-re, he wants to say "those letters which are common to the words 'ladder' and 'stairs'" and he writes $A \cap B = \{a,r\}$. In other words there are two letters which appear in both words.

And if he does the same thing for the second line of the riddle-me-re, he writes $C \cap D = \{a,p,e,s\}$. That is there are four possible letters he can choose.

Suppose, on the other hand, he wishes to indicate which different letters are necessary to write 'ladder' and 'stairs,' he writes $A \cup B$. In this case $A \cup B = \{l,a,d,e,r,s,t,i\}$ and similarly $C \cup D = \{a,p,l,e,s,r\}$.

Got it? Then try and decode this message. (You may have to re-arrange the letters in each word to do so). If you find you really have 'got the message' then code one of your own and send it to another member of your form.

1. $A = \{m,o,u,s,e,y\}$, $B = \{y,o,u,t,h\}$, $A \cap B = \quad ?$
2. $C = \{h,a,v,e,n\}$, $D = \{s,h,a,v,e,d\}$, $C \cap D = \quad ?$
3. $E = \{c,r,o,w,n,e,t,h\}$, $F = \{l,o,w,i,n,g\}$, $E \cap F = \quad ?$
4. $G = \{p,l,o,d\}$, $H = \{c,o,m,e,t\}$, $G \cup H + e = \quad ?$
5. $I = \{h,e,r,m,i,t\}$, $J = \{h,e,a,t\}$, $I \cap J = \quad ?$
6. $K = \{f,i,t\}$, $L = \{s,t,i,r\}$, $K \cup L = \quad ?$
7. $M = \{e,a,t\}$, $N = \{g,a,s\}$, $M \cup N = \quad ?$

S.T.P.

SEQUENCES

Sequences submitted by Mrs. G. Beard, Oastler College, Huddersfield.

Continue these sequences

1. 1, 2, 3, 4, 10, 11, –, –,
2. 1, 10, 11, –, –, –,
3. 1, 2, 3, 4, 5, 6, 7, –, –, –, –, 10, 11, –,

State the scales of notation.

15961 31854 34754 64955 69781 03829 30971 64651

INITIAL ALGEBRA 2

Some years ago I found the following on a price ticket:

IN/Y + CT/A = 72s. 3d.

The first letter group is the cost and the second group represents the purchase tax, which is a percentage of the cost such as $66\frac{2}{3}$, 50, $33\frac{1}{3}$ per cent. or a similar fraction.

Can you see why I concluded that someone in this shop is weak at spelling?

Submitted by Mr. G. Edgcombe, Plymstock Comprehensive School.

FUN WITH NUMBERS No. 8

Express the number 10 by using five 9's and find four different methods of doing this. R.H.C.

CAN YOU UNDERSTAND THIS?

A body by describing a kind of spiral might descend towards a revolving globe so that its apparent motion with respect to a point on the surface might be a straight line tending to the centre. (LEONARDO DA VINCI, 1510 A.D.)

THE ETERNAL TRIANGLE

An equilateral triangle of side 7 units is divided into equilateral triangles of unit length. How many triangles are there in the figure? R.H.C.

SENIOR CROSS FIGURE No. 47

1	2	3		■	4	
■	5		■	6		■
7			■	8		9
	■	10			■	
11	12		■	13	14	
■	15		16			■
17		■		■	18	

Ignore decimal points and work to the required degree of accuracy.

CLUES ACROSS:

1. The largest angle in degrees and minutes, of a triangle with sides 5.42, 7.63, and 6.21 units.
4. $\sqrt{2209}$.
5. $\sqrt{441}$.
6. The diagonal of a rectangular block 3 by 4 by 12 units.
7. The reciprocal of 14.
8. $\sqrt[3]{120}$.
10. 5 cubed.
11. $\sqrt{273}$.
13. $\sqrt{570}$.
15. $\sqrt[3]{26}$.
17 and 18. The smallest angle of the triangle in 1 across.

CLUES DOWN:

2. 11 squared.
3. π.
4. The longest stick, in inches, that will fit into a box 25, by 30 by 20 inches.
6. Area, in square inches, of a circle of diameter 4.3 inches.
7. The length of the hypotenuse, in cm., of a triangle whose other sides are 5 and 6 cm.
9. $\sqrt[3]{59}$.
12. $\sqrt{3894}$.
14. $\sqrt[3]{44}$.
16. 4^3.

B.A.

12424 70141 21478 05734 55105 00801 90869 96033

The sportsman need not be a student of Euclid to be aware of the difference in physical form between the long distance runner and the shot putter; or between the full-back and the outside-right. However, he may not realise the mathematical reasons for these disparities.

The athlete's energy depends on a continuous supply of oxygen which reaches the tissues via the surface area of the lungs. The dissipation of heat similarly depends on a surface area, the skin. In contrast, power is a function of a solid, the muscle bulk. A slight physique has a greater surface area in proportion to body mass than a sturdy physique. Thus the former is better adapted for endurance activities. Over a short distance the bulkier physique has the advantage. Not surprisingly, the heavier crew is generally favoured to win the Boat Race.

Large animals, with a smaller surface area in proportion to body mass, can more easily retain body heat than small animals. It follows that wild life in Polar regions partly depends on its size for survival and is rarely small. Carl Bergmann recognised the mathematical reasons for animal size variations.

Considering an animal, plant or engineering structure, it is evident that the weight is proportional to the volume. Yet the strength is proportional to the area of cross-section of the supports. Thus, of two geometrically similar structures, the larger is the weaker. Large animals require thicker skeletons for support. Large fruits are often produced at low levels for ground support.

Discovering that stalling-speed varies as the square root of the linear dimension, one can appreciate why large birds require a running take-off and why the ostrich does not take to the skies.

A detailed account on the practical applications of similarity and proportion is given in that ever useful volume 'On Growth and Form' by D'Arcy W. Thompson.

D.I.B.

02763 47870 81081 75450 11930 71412 23390 86639

38339 52942 57869 05076 43100 63835 19834 38934