OPTICAL TOYS

The retina of the eye retains an image for about one twelfth of a second after it has disappeared ; this defect of the eye enabled the film industry to present a series of still pictures in rapid succession to an audience who saw them as a continuously changing picture. This persistence of vision also enables cartoon films to instruct as well as to entertain.

One of the earliest optical toys using this property of the eye was a ZOETROPE, see Fig. 1. This consisted of a cylinder with slots cut at regular intervals around it. Inside was placed a length of card, see Fig. 2, on which a series of pictures was drawn. Each picture showed an action in a slightly later position. The cylinder was rotated and the action appeared to be continuous. This toy was used mainly to show a repeated action in which the penultimate position was one step from the beginning of the repeat of the action.

Optical Toys

Fig.1

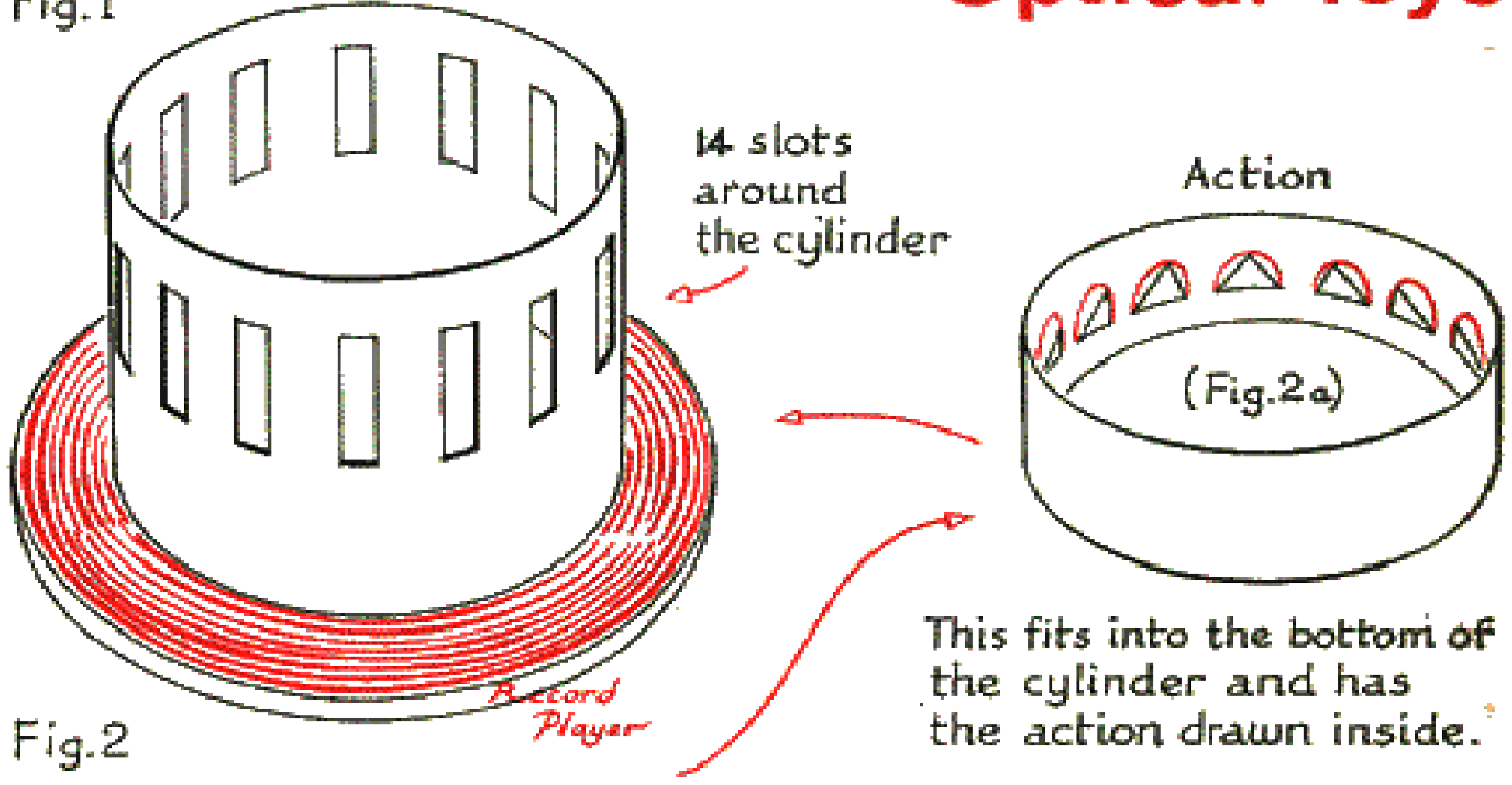


Fig.2

14 Pictures

Take a piece of sheet metal, or thick card, 28½ inches by 4 inches. Cut 14 slots 1½ inches by ¼ inch at intervals of 2 inches along the length of the strip, each slot starting from the centre line and lying in the top half of the strip. Join the ends to form a cylinder 28 inches in circumference. This forms the basis of the Zoetrope. The action consists of 14 diagrams drawn on thin card, 28 inches by 2 inches, and spaced 2 inches between the centres of the figures. This is placed inside the cylinder and the whole is mounted on a record player turntable. When the turntable rotates at 78 r.p.m., the persistence of vision of the eye gives the illusion of continuous motion when the action is viewed through the slots of the cylinder.

A FLICKER-BOOK can be used to simulate continuous action which requires a greater number of pictures or which is not repeated. Up to fifty pages may be used in a single sequence. A suitable mathematical subject

Continued on page 410

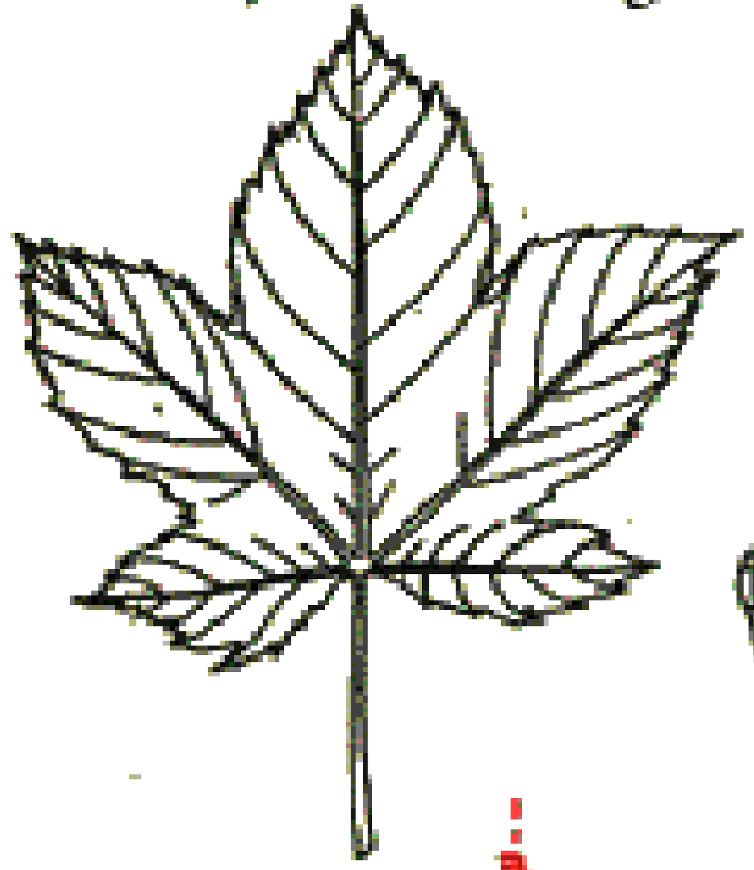
mathematical pie

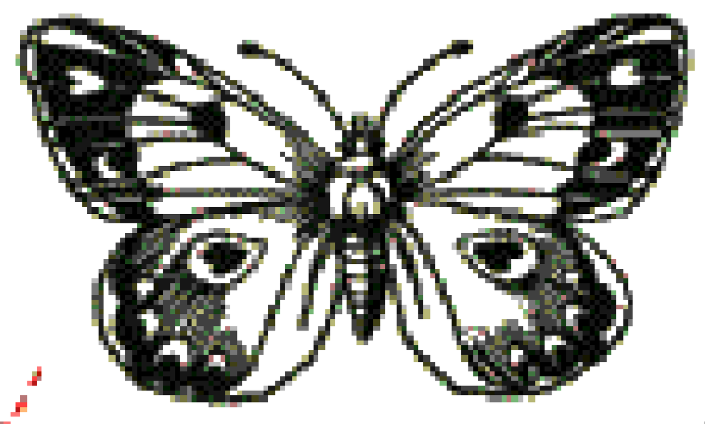
No. 52 *Editorial Address:* 100, Burman Road, Shirley, Solihull, Warwickshire, England **OCTOBER, 1967**

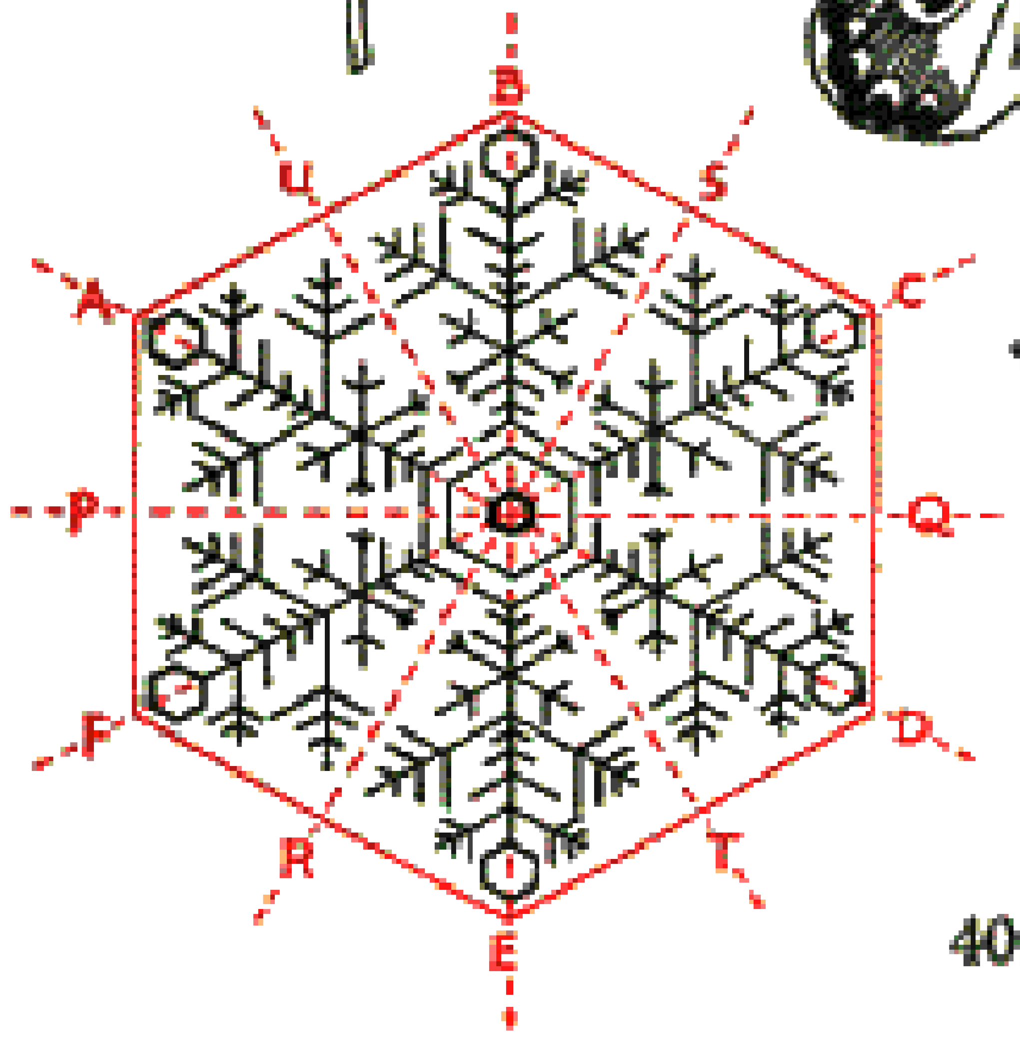
SYMMETRY

Some figures may be transformed in such a way that they appear to have remained in the same position. The figures can be transformed into themselves and are said to be symmetrical.

The snow crystal (not to be confused with the snow flake) shows two types of symmetry found in its basic pattern, the regular hexagon. By rotating about the centre through multiples of 60°, the hexagon will appear to be unchanged. For example, rotating through 120°, the position will appear unchanged, but letter A will be in position C, B in position D, and so on. Thus the hexagon has rotational symmetry. Drawing a diagonal (say AD) it is seen that if the figure is folded along this line, one part ($ABCD$) fits exactly over the other ($AFED$). AD acts as a mirror with $ABCD$ and $AFED$ as object and image. Each diagonal similarly produces a reflection



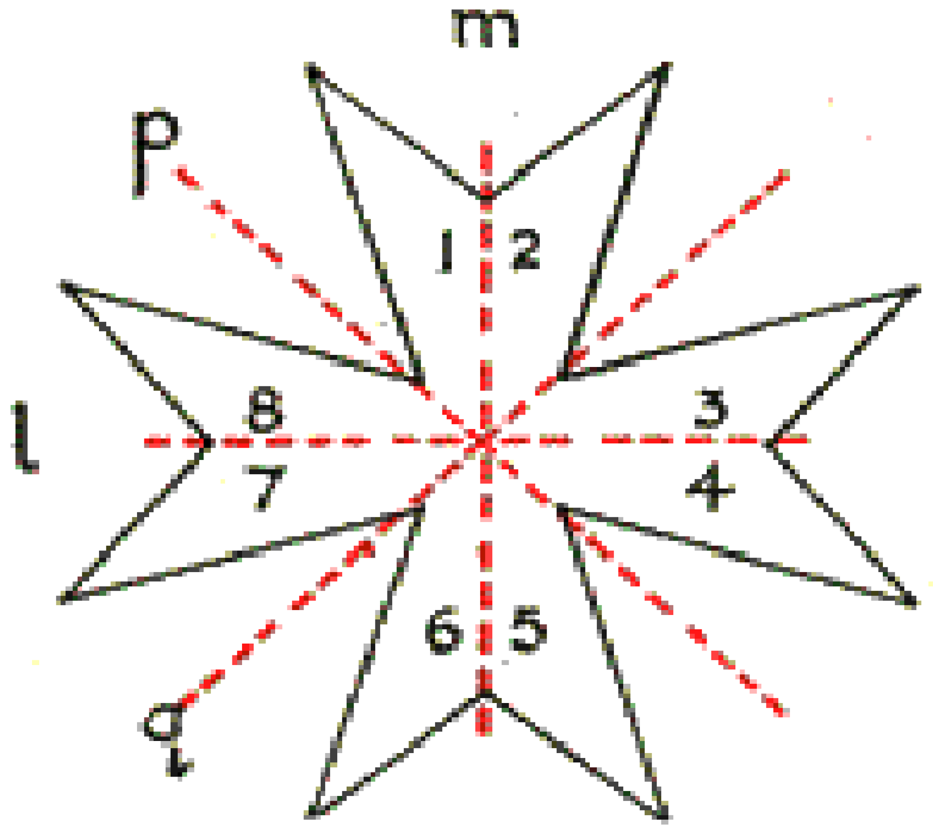




and the same is true for each of the perpendicular bisectors (*PQ*, *RS*, *TU*) of the sides. Thus the hexagon also possesses reflective symmetry and the lines which behave as mirrors are known as reflective axes.

Considering the drawings of the leaf, butterfly and Spruce tree, it is quite easy to discover that each has one axis of reflective symmetry. The starfish obviously has rotational symmetry, but can you find all its axes of reflective symmetry?

By further experiment, it is possible to discover more properties of symmetrical figures. As an example, the Maltese Cross has been chosen.



Considering rotation in a clockwise direction, "*a*" represents a turn of 90°, a^2 a turn of 180°, and a^3 a turn of 270°. Another requirement is a symbol which means 'no movement' or 'leave alone': for that, *I* is used. Hence, a^4 is unnecessary. Considering reflection, the four axes of reflection are labelled, *l*, *m*, *p*, *q*.

The table is used to find one operation in each case which is equivalent to a combination of any two operations. The top row is taken first and the left column second.

For example, a reflection in "*l*" followed by a rotation "a^3" gives a position which could have been obtained with the single operation "q". In the same way try to fill in the blanks in the table. The numbers on the cross are to help you identify the new positions.

Try to find the special properties of this table and compare them with those for a table of addition in clock arithmetic (issue No. 34).

More can be read about symmetry in issues No. 38 and No. 39. D.I.B.

1st operation (columns); 2nd operation (rows)

	I	a	a^2	a^3	l	m	p	q
I								
a				I				
a^2								
a^3					q			m
l								
m								
p								
q			p					

STAMP CORNER NO. 23

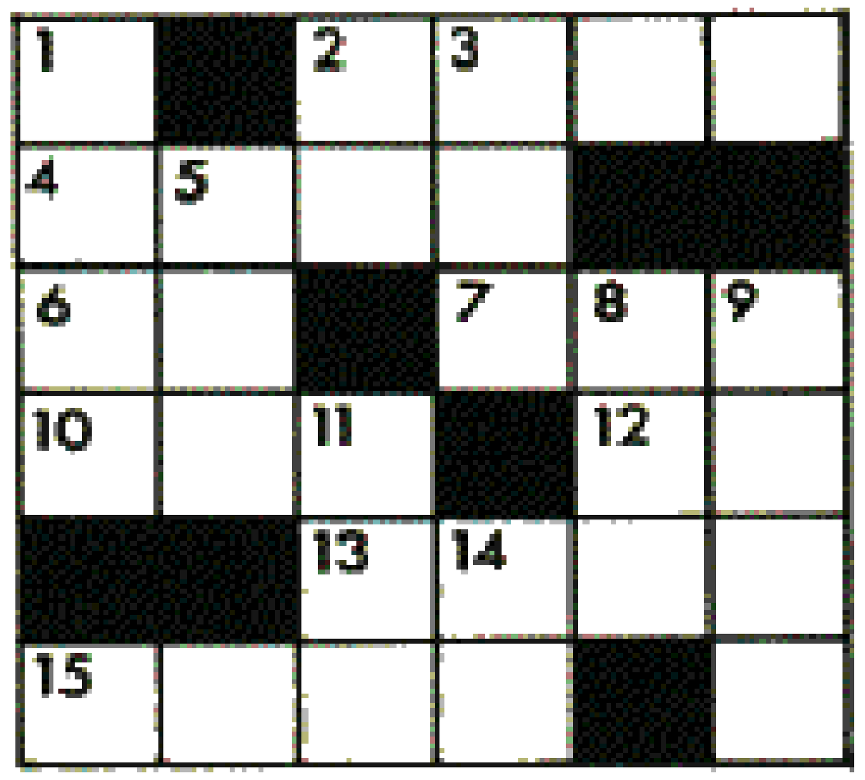
Leibniz (1646-1716) was Newton's contemporary and independently developed the ideas of the differential calculus. He introduced the dx/dy notation and the rules for differentiation of products and quotients. In algebra, he began the study of determinants. One of the oddities of Mathematical history is Leibniz's use of "numbers to stand for letters". He used 11, 12, . . . 21, 22, . . . to stand for the coefficients in simultaneous equations. This notation he later changed to the now usual a_{11}, a_{12}, etc. C.V.G.

FRACTION FAULT

A student was asked to find 12/13 of a certain number but made the mistake of dividing the number by 12/13 and got an answer which was 25/162 greater than the correct answer. What was the correct answer? R.H.C.

JUNIOR CROSSFIGURE NO. 45

Ignore decimal points and work to the appropriate number of figures.



CLUES DOWN:

1. Next two terms in the sequence 16, 25, 36, 49, . . .
2. Simple interest on £47. 10s. at 5% per annum for 8 years.
3. One Gunter's Chain link in inches.
5. Area of a square of which the perimeter is 56 units.
8. Smaller angle between the hands of a clock at 6.15.
9. Number of revolutions of a 14-inch diameter wheel in 1 mile.
11. £1. 8s. 2d. as a decimal of £5.
14. Number of faces of a regular polyhedron which has 12 vertices and 30 edges.

CLUES ACROSS:

2. Cubic inches in a cubic foot.
4. Product of the "teens" primes.
6. Area of a quadrilateral with vertices (0, 0), (15, 3), (12, 10), (2, 7).
7. A number whose digits have a sum of 12.
10. The golden ratio.
12. Area of a square on the shortest side of a right-angled triangle with other sides 19½ and 17½ units.
13. $2^{13}+2^{8}-2$.
15. Area, in square inches, covered by £3. 10s. 0d. of halfpennies. $\pi=22/7$.

D.I.B.

SOLUTIONS TO PROBLEMS IN ISSUE No. 51

WHERE ARE THE TOWNS? The towns may lie in a semicircle of radius 3 miles with centre at the road junction, unless the road doubles back when the locus is a full circle.

WHEN PAINTING MY HOME. The ladder was 13 feet long.

100 TREES. The bullet would miss every tree as $\sqrt{3}$ is an irrational number.

PASCAL.

The name Pascal can be spelt out in 186 ways. If we take one sixth of the diagram, we have

```
          P                                  P
        a   a                              1   1
      s   s   s                          1   2   1
    c   c   c   c                      1   3   3   1
  a   a   a   a   a                  1   4   6   4   1
1   1   1   1   1   1              1   5  10  10   5   1
```

If we replace each letter by the number of ways of reaching it from the P we get the table on the right. The sum of the figures along one side of the hexagon is 1 + 5 + 10 + 10 + 5 + 31. Why not the last 1? Hence the total number of ways is 6 × 31 = 186. R.M.S.

HAVE YOU EVER BEEN ADD?

If the first number is *a* and the second number is *b*, then the numbers are a, b, $a+b$, $2+2b$, $2a+3b$, $3a+5b$, $5a+8b$, $8a+13b$, $13a+21b$, $21a+34b$ giving a total of $55a+88b$ which equals $11(5a+8b)$. Did you notice the Fibonacci numbers in the sequence? See issue No. 48.

SENSE OF TOUCH. Eight circles can be drawn.

JUNIOR CROSSFIGURE No. 43

Across: 1. 3142; 5. 8649; 6. 100; 8. 19; 9. 405.

Down: 1. 381; 2. 16024; 3. 440; 4. 29; 7. 39; 8. 15.

SOLUTIONS TO PROBLEMS IN ISSUE No. 52

FRACTION FAULT. The correct answer was 8/9.

ALL SQUARE. The product of two squares is itself a square. Although
$(4-1)(9-1) = 24 = 25 - 1$; $(4-1)(16-1) = 45$ which is not one less than a square.
$(4+3)(9+3) = 84 = 81 + 3$; $(4+3)(16+3) = 133$ which is not 3 more than a square.
$(x^2+c)(y^2+c) = z^2+c$ only if x, y differ by unity.

A SLIM TIME: The girl slimmed for six weeks.

WHY GO DECIMAL? Lemon

FOR AMUSEMENT ONLY! $\frac{1}{10}$ if the numbers are uniformly distributed on the cards.

B.A.

along the stall to ensure that they do not stick in the same card? Each row occupies $\frac{1}{5}$ of space, or $\frac{4}{5}$ for all four rows, and $\frac{20}{49}$ of each row is occupied by cards, *i.e.*, $\frac{4}{5}$ of $\frac{20}{49}$ or approximately $\frac{1}{3}$ of the area is occupied by cards. This means that roughly 1 dart in 3 will stick in a card, or, as mathematicians say, the probability of success is $\frac{1}{3}$. If you throw two darts, the probability of both in cards is $(\frac{1}{3})^2$, and for three darts only $(\frac{1}{3})^3$ *i.e.*, once in 27 attempts. No wonder the prizes look a bit dusty! However it can be shown (try it!) that $\frac{5}{9}$ of the time one or two darts will be successful, which is more than enough to encourage you to have another try—at 6d. a time! Better try BINGO!

If there are 40 Bingo cards on a stall each with 5 columns of 5 numbers, fig. 11, and you can win with a line, a column or a diagonal, what is the probability of your winning a given game, if the balls are numbered 1 to 99?

If you find any interesting mathematics in any other machines or side shows, please let the Editor have the necessary details. S.T.P.

ALL SQUARE

4 and 9 are squares and 4.9=36, which is also a square. Does this work for any two other squares?

Take one off 4 and 9, and multiply the results together. Have you any comment to make about the relationship and does it work with other squares?

Add 3 to both 4 and 9 and multiply the results together. Does this work with other squares? R.H.C.

Continued from page 412

Fig. 3

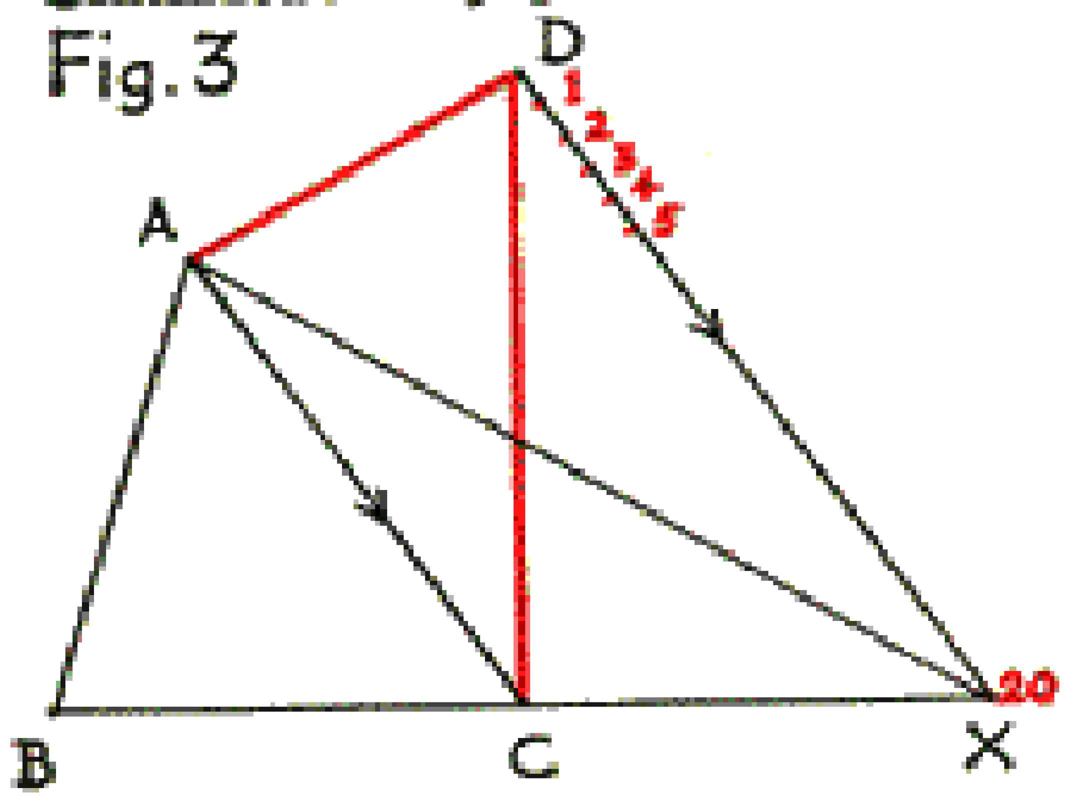


Fig. 4

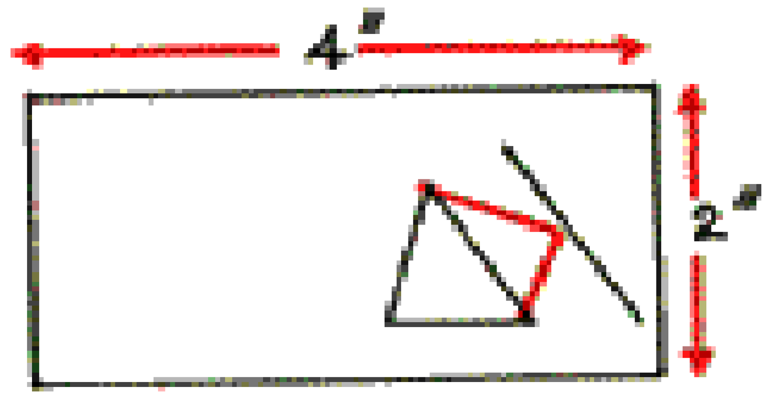
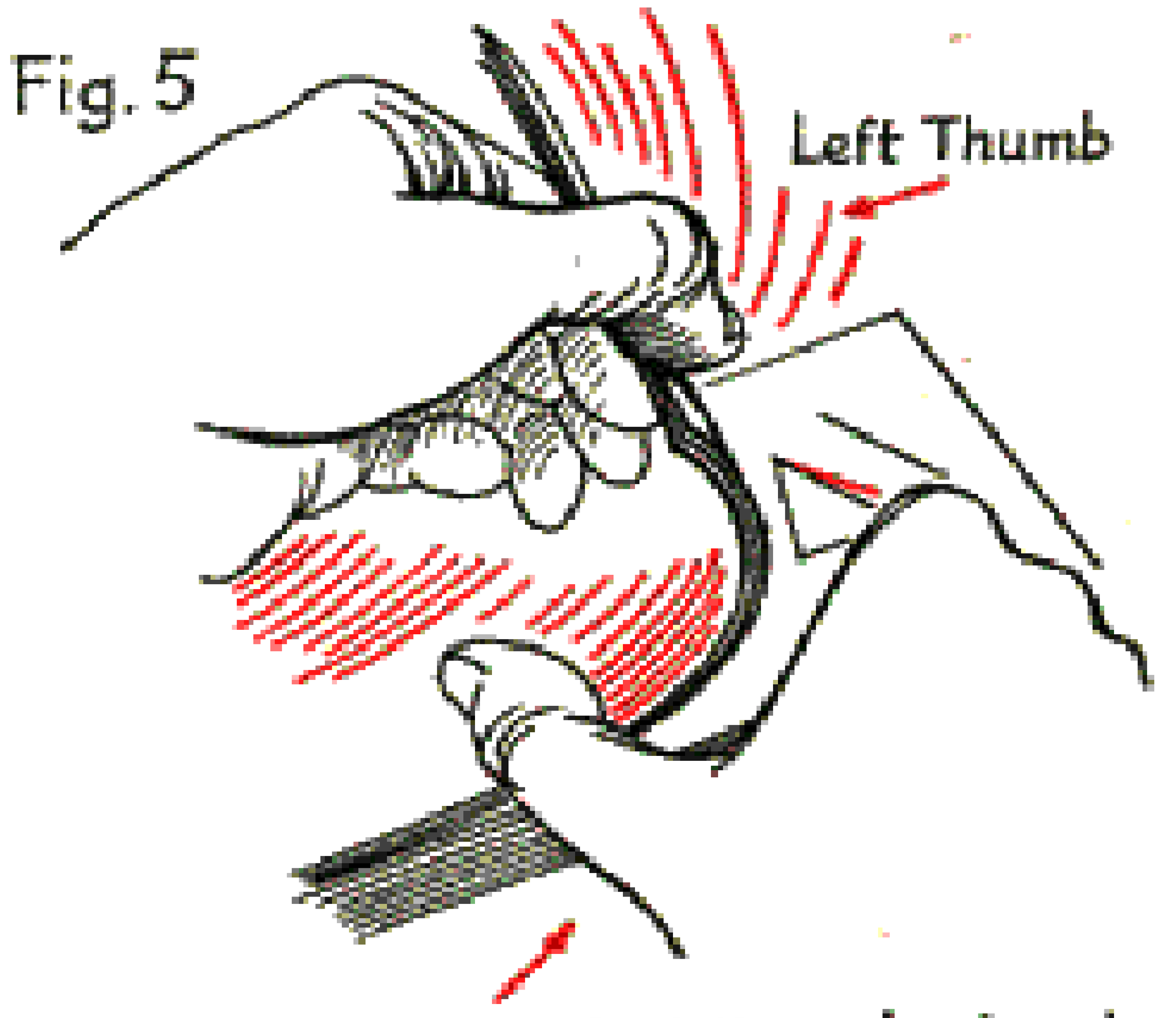


Fig. 5



Right hand holding the book with thumb on top.

for a Flicker-book is the transformation of a quadrilateral into a triangle of equal area, see Fig. 3, 4, and 5. Using thin cards, 4 inches by 2 inches, draw accurate diagrams so that (i) the triangle *ABC* is in exactly the same position on each card, (ii) the point *D* moves in 20 steps along the line *DX*, so that the triangle *ADC* gradually becomes the triangle *AXC*. A typical card is shown in Fig. 4. Finally, the cards are stapled together to form a book, the first position of *D* being at the bottom and the subsequent positions being arranged in order above it. The book is held by the right hand near the staples and the leaves are flicked from back to front using the thumb of the left hand, see Fig. 5. The quadrilateral *ABCD* appears to deform continuously into the triangle *ABX*. B.A.

A SLIM TIME

Believe it or not, but Cutie is rather sensitive about her weight, though at eight stone she should worry! Periodically, she goes on a week's diet, during which time she sheds three pounds. Not surprisingly, the effect is not very pronounced as she gains half a pound a week when not dieting. Last year, for example, her nett gain was five pounds from which you can rapidly deduce how many weeks, of the fifty-two, were spent longing for her favourite chocolate. S.T.P.

WHY GO DECIMAL?

Suggested by Mr. A. Steel, Harold Cartwright School, Solihull.

Have you ever stopped to think of the many different number bases we use? 2 pints=1 quart, 3 feet=1 yard, 4 quarts=1 gallon, 7 days=1 week, etc.

Each of the numbers in the following list is used in this way. Can you think of the context? 2, 3, 4, 7, 8, 10, 12, 14, 16, 20, 22, 24, 36, 52, 60.

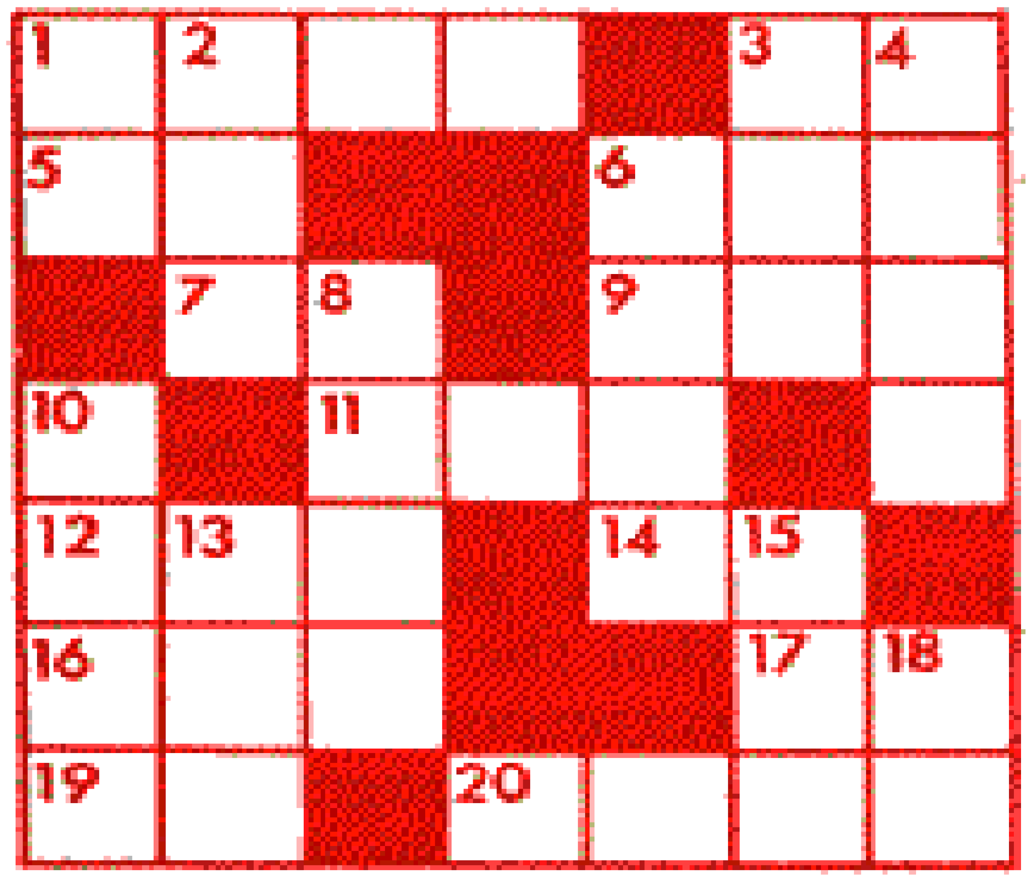
If we leave out 10 (because we shall continue to need this) and 12 (because it is useful), what is the total? Reverse the digits and subtract the original total. Reverse the difference and add the original difference. Now let's use our 12 and multiply the last answer by it.

If we are to enter the Common Market our result must be of use to Mlle. Yvonne as well as to George next door. Use the code

MLLEYVONNE
0 1 2 3 4 5 6 7 8 9 to translate your final answer. R.M.S.

SENIOR CROSSFIGURE NO. 48

Ignore decimal points and work to the appropriate degree of accuracy.



CLUES ACROSS:

1. tan 240°.
3. $(\frac{1}{8})-1\frac{1}{2}$.
5. Degrees difference in longitude between two places where local times differ by 3 hr. 4 min.
6. Area between the *x*-axis and the curve $y=x^2+x-6$.
7. log_2 104.
9. 7 radians in degrees.
11. Minimum value of $3x^2+2x-7$.
12. $\sqrt{656100}$.
14. Half an interior angle of a regular twenty-sided polygon.
16. 56th term of the series 35, 48, 61, 74, . . .
17. The principal in £ which, in 4 months, gives 10s. 2d. simple interest at a rate of $2\frac{1}{2}$% per year.
19. Velocity at 2 sec., when the distance is given by $s=16t^2-10t$.
20. Solve, giving x first
$8643x+1357y=23643$
$1357x+8643y=16357$.

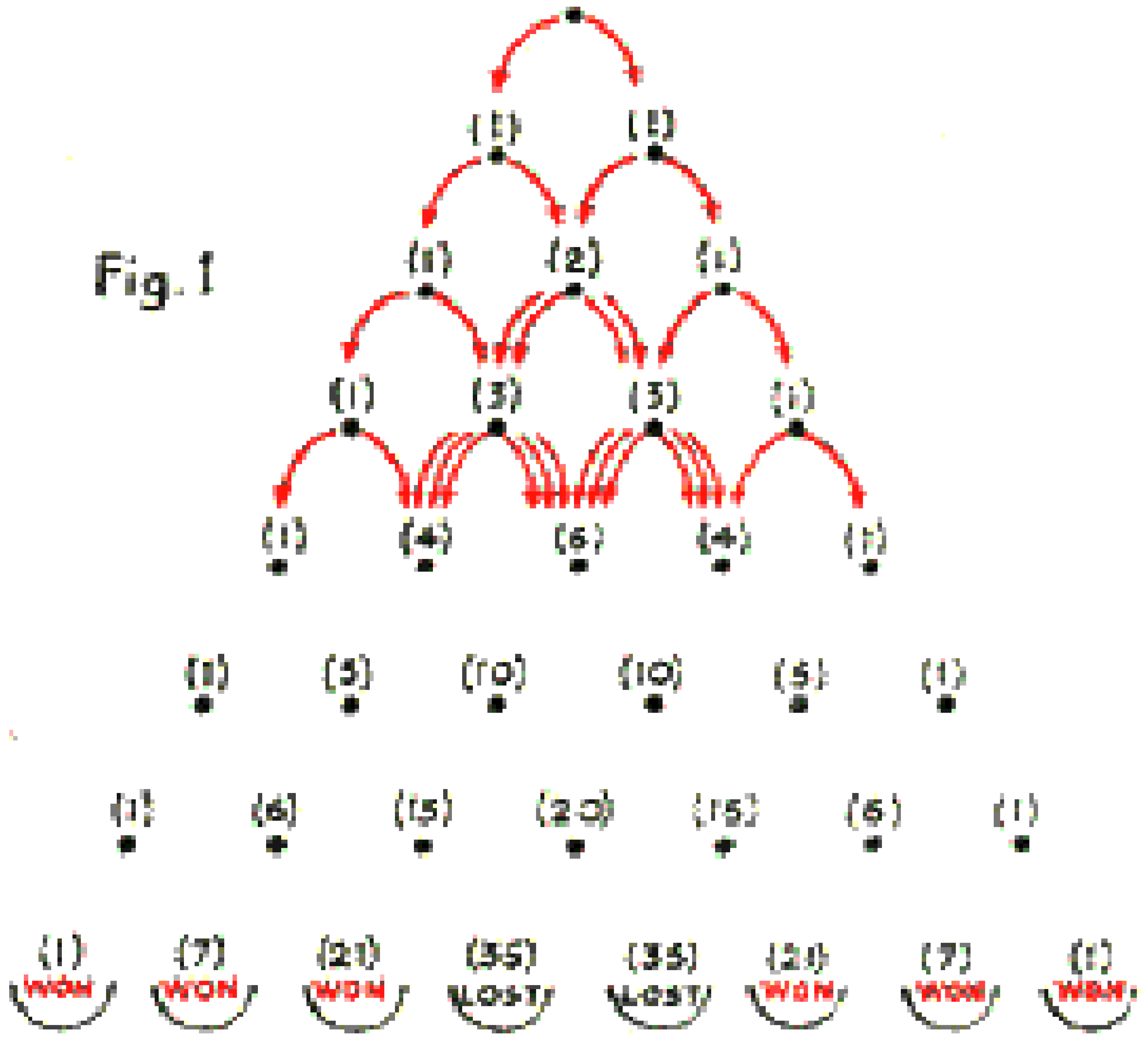
CLUES DOWN:

1. Gradient at (2,9) of $y=3x^2+2x-7$.
2. Reverse the sum of the roots of $3x^2+2x=7$.
3. Sum to infinity of the series 300, −150, 75, . . .
4. Number of ways, reversed, of forecasting results of 7 football matches if each can end in a 'home', 'away', or 'draw'.
6. Speed (to nearest knot) and bearing of a ship which steers at 16 k. on a bearing of 65° in a current flowing at 12 k. due North.
8. Circumference (to the nearest 100 miles) of 72° latitude, taking the earth's radius as 3,960 miles.
10. tan θ as a decimal when cos θ=8/17.
13. Area enclosed by $x^2+y^2=49$.
15. If three consecutive multiples of 7 add up to 504, find the least of the three.
18. The radius of a circle in which a 24 inch chord is 9 inches from the centre.

D.I.B.

For Amusement Only

Fig. 1



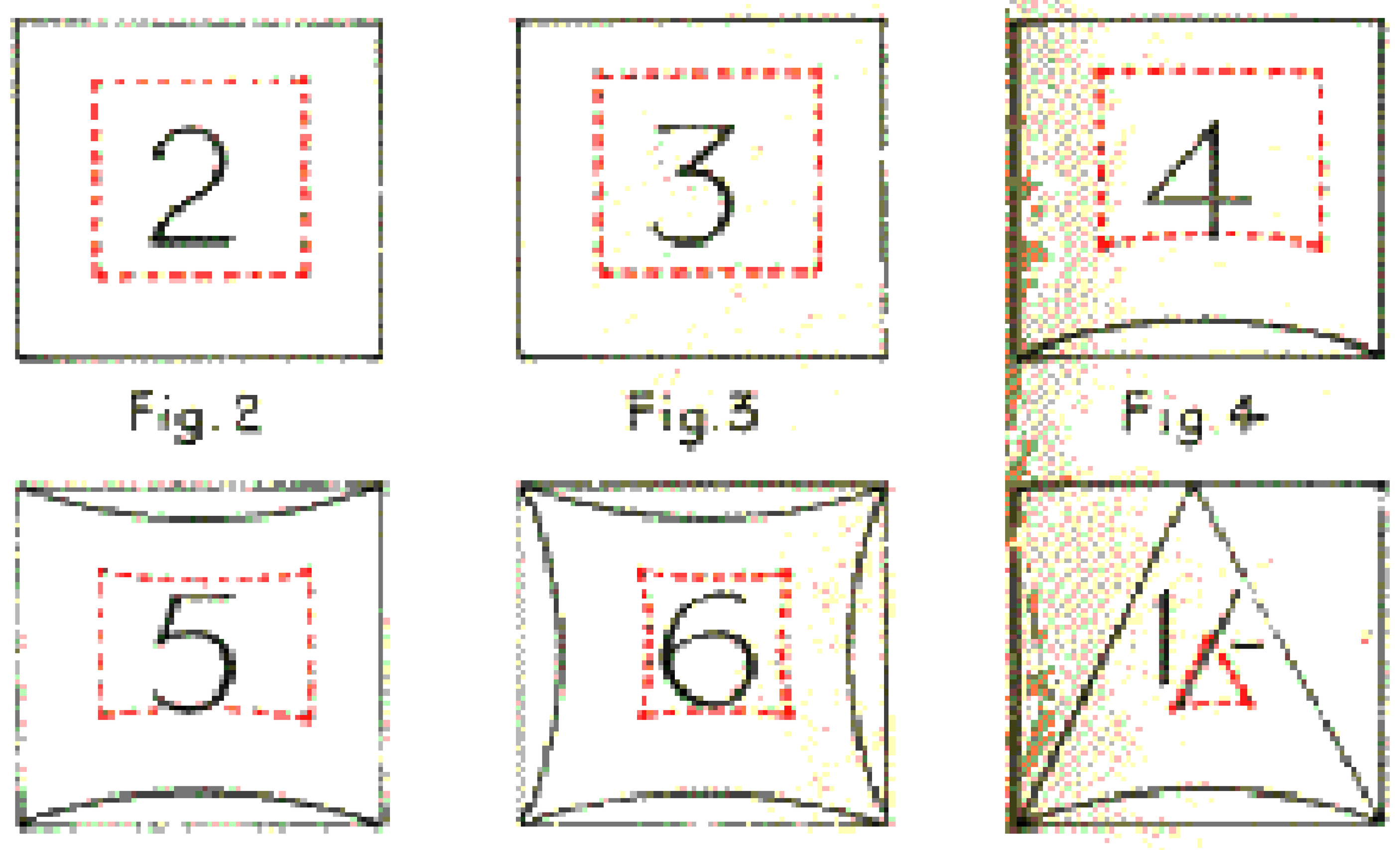


Fig. 2 Fig. 3 Fig. 4

Fig. 5 Fig. 6 Fig. 7

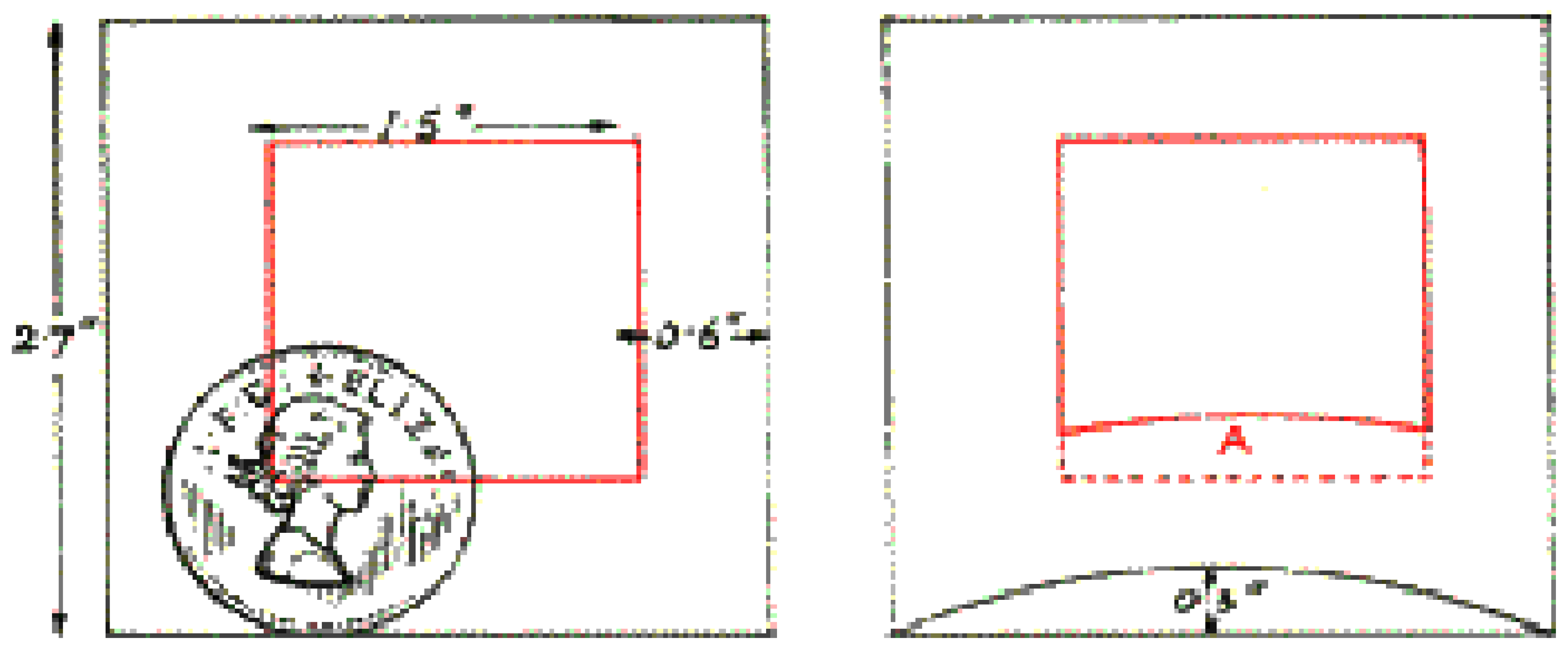


Fig. 8 Fig. 9

The coloured lines show the boundaries of the areas where the centre of the penny can land.

Slot machines and side shows at the fair, or in the amusement arcade at the seaside, fascinate most people, the prospect of winning a ' fortune ' being irresistible. At the same time they can be extremely interesting to a mathematician. Take, for example, the machines where a ball whizzes round at the flick of a handle, chatters between rows of nails and disappears into a hole—usually marked ' LOST.' The interesting part of the machine is shown in fig. 1. The ball first strikes the top pin and can turn either left or right. Whichever pin it reaches in the second row it again has two choices. Thus there are two ways in which it can arrive at the centre pin of row three, but only one way to each of the outer pins. If a large number of balls passed through the machine, approximately 25% would arrive at each of the outer pins and 50% at the centre pin. Whenever the ball strikes a pin there is the same choice, and by tracing out the paths shown on the diagram it will be seen that the number of paths leading to the pins in the fourth row are 1, 3, 3, 1 respectively. As all the paths are equally likely, the balls arriving at the pins will be in these same proportions. It will soon be apparent that the number associated with each pin is found by adding the numbers corresponding to the pins immediately above. (We have met this array before, see pages 340, 365, 370 in issues 43 and 47). Consider row eight. At first sight with two ' LOST ' and six ' WIN ' cups you seem to be on to a good thing, but look carefully at the numbers—a total of 70 for the ' LOST ' but only 58 for the ' WIN ' cups. Perhaps you had better roll pennies instead.

At a well-known stately home, the side show at which you roll pennies has a board divided into squares (with sides 2·7 inches long) marked as shown in the figures 2 to 7. Let us consider, in detail, the square shown in fig. 8. For a penny to land without touching the black line, its *centre* must lie somewhere inside the coloured square, taking the diameter of a penny to be 1·2 inches. As the ratio of the areas of the red and black squares is $(1{\cdot}5)^2 : (2{\cdot}7)^2$ or slightly under 1 : 3, the centre of the penny will be in a winning position, on average, 1 roll in 3. The square in fig. 2 is just about fair, whilst that in fig. 3 is to your advantage. Now consider fig. 9 (which is an enlargement of fig. 4) with the winning area again shown in red.

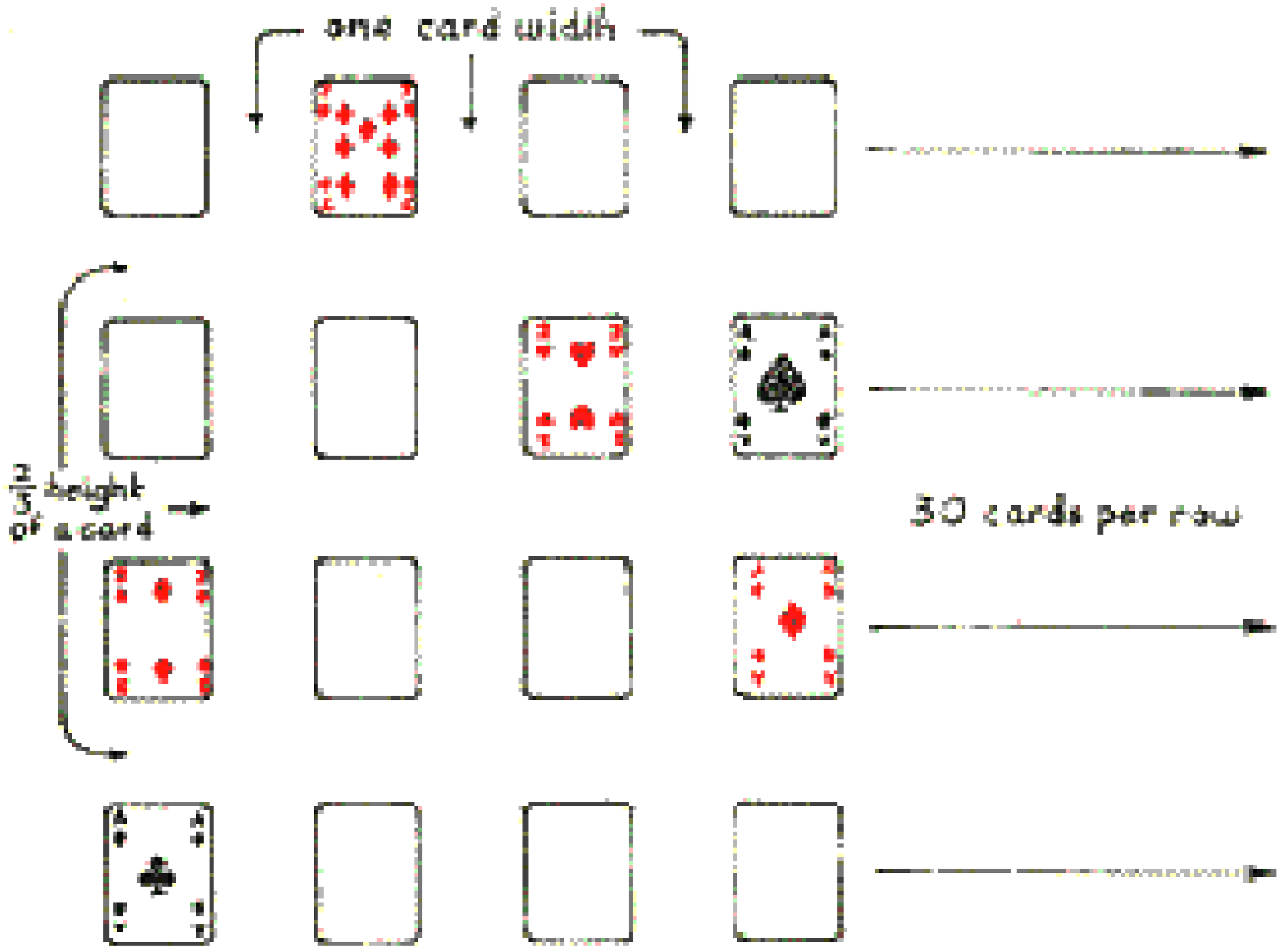


Fig. 10

1	22	43	61	82
2	24	47	65	84
5	36	50	67	90
17	38	55	73	95
19	40	57	78	99

Fig. 11

The area *A*, which is the amount now removed from the original winning area, can be shown to have an area of approximately $\frac{3}{8}$ of a square inch. (This is a good exercise on the areas of sectors and segments!). The ratio of the areas of the red and black areas is now approximately 1 : 4 and the odds are still slightly in your favour. For fig. 5 the ratio is 1 : 5 but for fig. 6 and fig. 7 approximately 1 : 8 and 1 : 90 respectively. Since there is only a small number of squares marked 6d. and 1/-, the board is designed to be slightly in favour of the proprietor (especially when the thickness of the lines is considered) but gives enough wins to encourage you to keep rolling. Who's for a game of darts?

Here four rows each containing 30 cards are spaced as in fig. 10. The object is to throw 3 darts and stick them into three separate cards. What is the probability of winning, assuming that you can throw straight enough to get the darts (*a*) in the area containing the cards and (b) sufficiently spaced