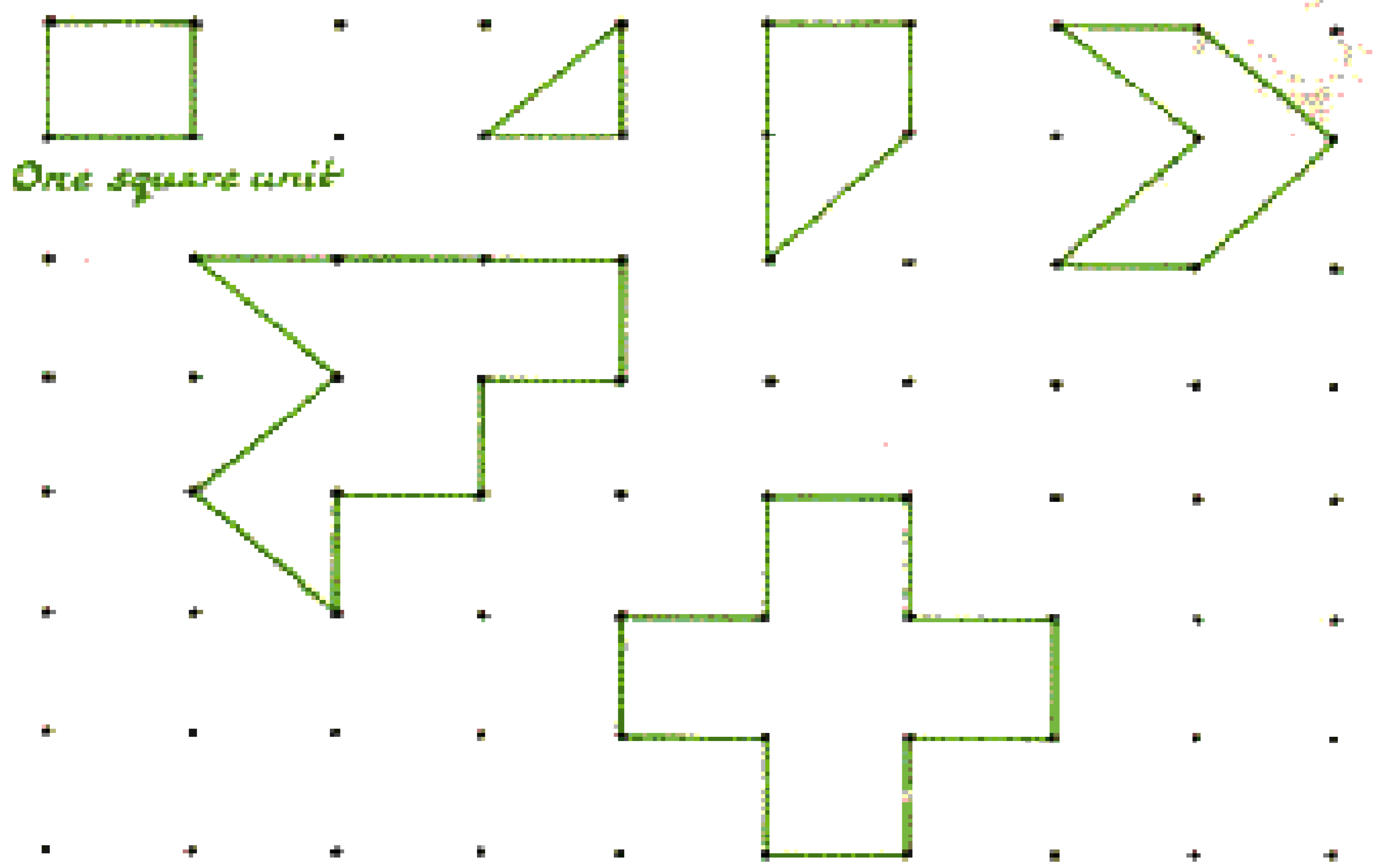
AN AREA PROBLEM



You are probably all familiar with area and how to find the area of well known shapes. Let us now look at a problem concerning area. Suppose we have a lattice of points as shown in the diagram and we call the area formed by joining four of the points to form a square, one square unit. We can now join points together to form some shapes according to the rule that no points are allowed inside the figure. Some examples are shown in the diagram. If we join three points together we find the area of the shape formed is half a square unit, four points joined give us an area of one square units and five points an area of one and a half square units. Make a table of your results and see if you can see any pattern in it. Can you write down a formula connecting the number of points used p and the area a?

No. of points used	3	4	5				
Area of shape	$\frac{1}{2}$	1	$1\frac{1}{2}$				

If you want to make the problem more difficult, investigate what happens if we allow points to be inside the shape. Make some shapes by joining four points together, enclosing one, two, three points, etc., as shown. Make a table of your results

No. of points on outside of figure	4	4	4	4	4	4
No. of points inside the figure						
Area						

Can you see any pattern in the table? Now do the same using five points on the outside of the figure. Repeat with six points, seven points, etc., until you can see a general pattern.

Can you give a formula connecting the number of points inside, the number of points on the boundary and the area?

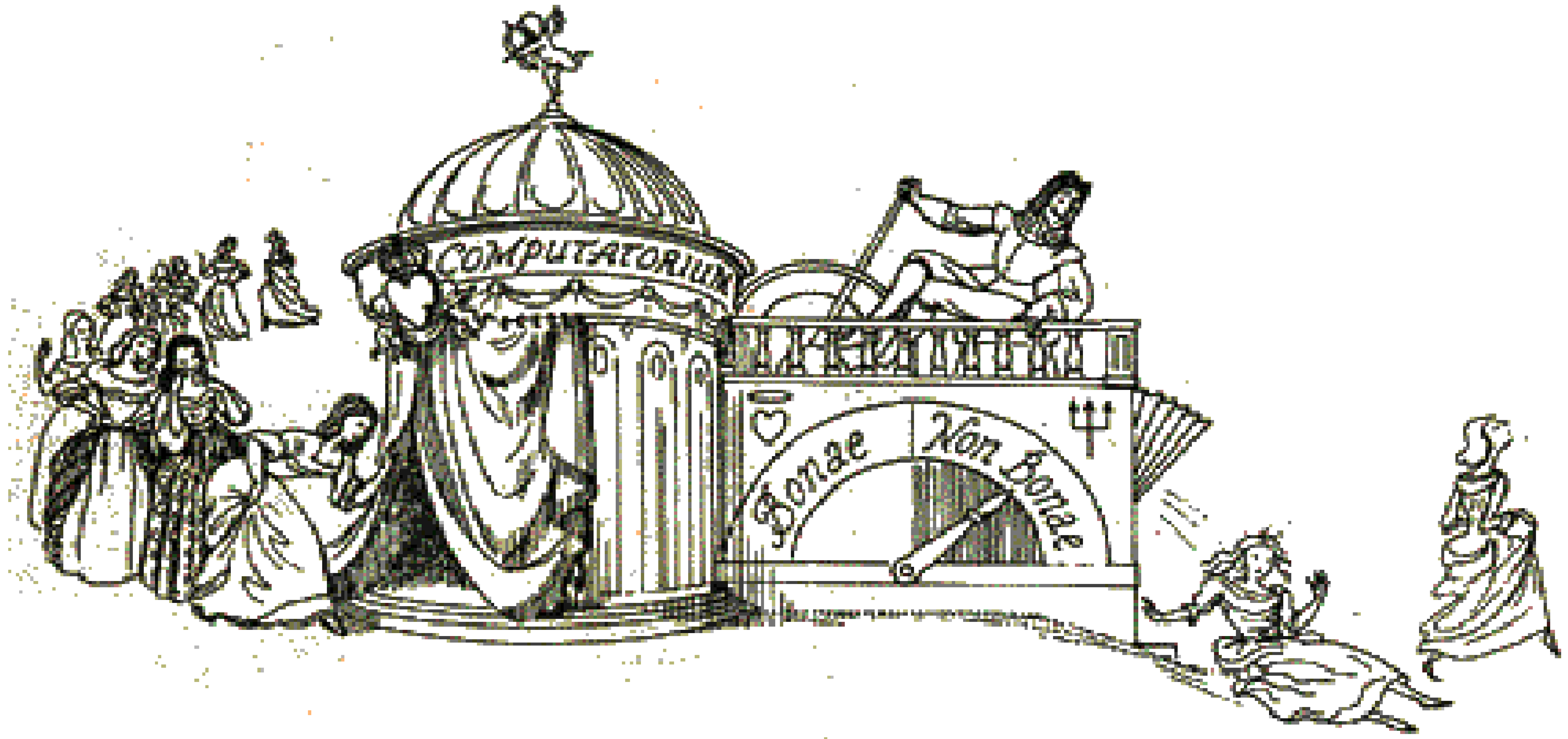
1. No points inside figure $n = 2a + 2$ n is no. of points, a is area.
2. Four points on outside $a = p + 1$ p is no. of points inside.
   Five points on outside $a = p + 1\frac{1}{2}$
   Six points on outside $a = p + 2$
   Seven points on outside $a = p + 2\frac{1}{2}$, etc.
   General pattern $a = p + \frac{1}{2}(n - 2)$
   $2a + 2 = 2p + n$.

A.W.B.

mathematical pie

No. 53 — *Editorial Address:* 100, Burman Road, Shirley, Solihull, Warwickshire, England — SPRING, 1968

WINE, WOMEN AND—



Kepler was born in 1571 and was one of the precursors of calculus. In order to compute the areas involved in his second law of planetary motion he had to resort to a form of integral calculus which he also applied to his " Stereometria dolicorum vinorum ", (Solid Geometry of Wine Barrels) in which he showed how to find the volume of 93 solids obtained by rotating segments of conic sections about an axis in their plane.

One report says that his second marriage was less fortunate than his first although he took the precaution to analyse carefully the merits and demerits of eleven girls before chosing the wrong one.

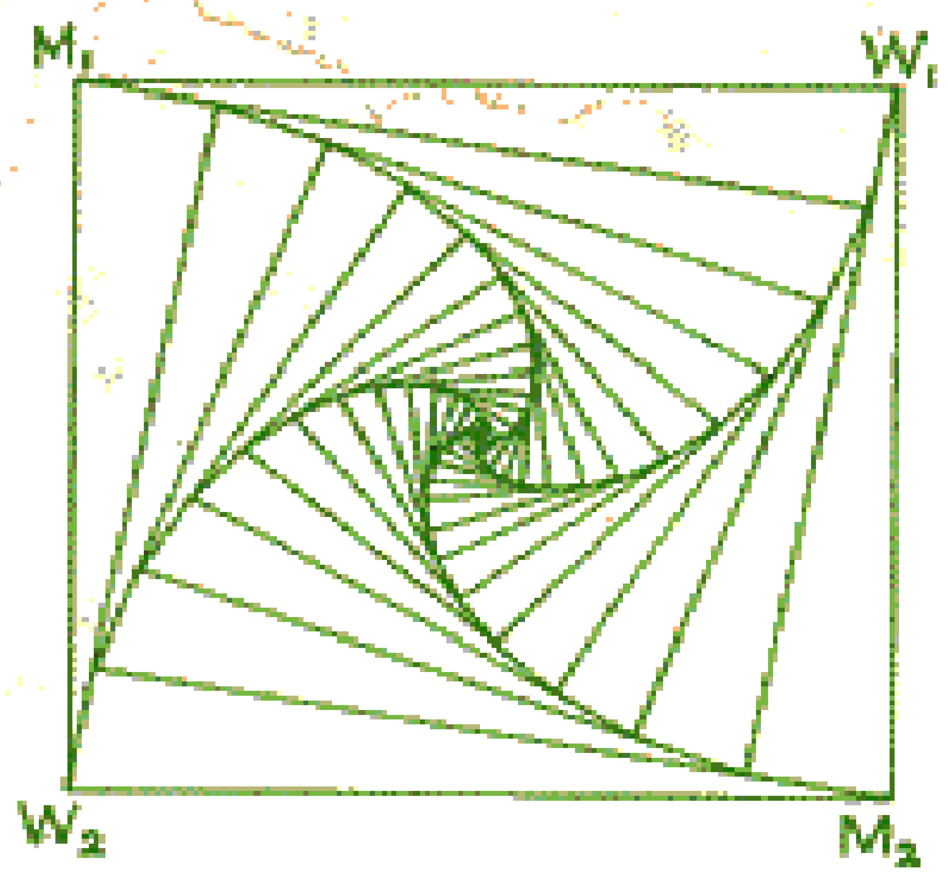
He died in 1630 while on a journey to obtain some of his long overdue salary.

R.H.C.

STITCHCRAFT

The last thing I expected when I took my case to be repaired was a discussion on prime numbers. 'You know', said the man skilfully stitching the leather, 'I've tried for years to find a formula for prime numbers, but I haven't found one better than Euler's n^2+n+41 which gives prime numbers for $n=0, 1, \ldots 39$. (Check it!) But I'll tell you what I have noticed. If I take any prime number greater than 3 and add or subtract one, one of the two numbers I obtain will be divisible by 6.' Can you see why?

S.T.P.



SQUARE DANCE

The formation for this dance is a 30-foot square with the two couples positioned at the corners, as shown. Following 'On your corners', M_1 moves towards W_1, who simultaneously moves towards M_2 and so on. In this way all the dancers move towards each other at the same speed until they meet. A star is then formed, but the problem is to find the distance moved by each dancer from the corner to the meeting point.

In a discussion which followed the suggestion of the above problem at an Editorial Board meeting, it was pointed out that the dancers would actually have to accelerate so much to retain their courses that they would eventually slip if they tried to follow the next dancer. They would therefore finish the dance by moving in a circle; this part of the problem is of interest to Sixth-form students.

D.I.B.

SUBSTITUTION

In the equation below, the letters X and Y each represent a number between 0 and 10. Find X and Y.

$$2(XYXYXY)=9(YXYXYX).$$

J.F.H.

POWERS

Work out the values of $0^3, 1^3, 2^3, 3^3, 4^3, 5^3, 6^3, 7^3, 8^3, 9^3$. Write down the terminal digit in each case. What do you notice? Does this happen for the fourth powers of the numbers and can you take this problem any further?

R.H.C.

SQUARE FROM A CIRCLE

What is the largest square which can be cut from a quadrant of a circle of diameter 8 inches?

D.I.B.

GROUP STRUCTURE

Divide the positive whole numbers into even and odd sets. Take any member of the even set and add it to any other member of the even set, the resulting number is a member of the even set. Take any member of the even set and add it to any member of the odd set, the resulting number is a member of the odd set. Take any member of the odd set and add it to any member of the odd set, the resulting number is a member of the even set. These results can be represented by the table 1.

+	e	o
e	e	o
o	o	e

TABLE 1

×	+	−
+	+	−
−	−	+

TABLE 2

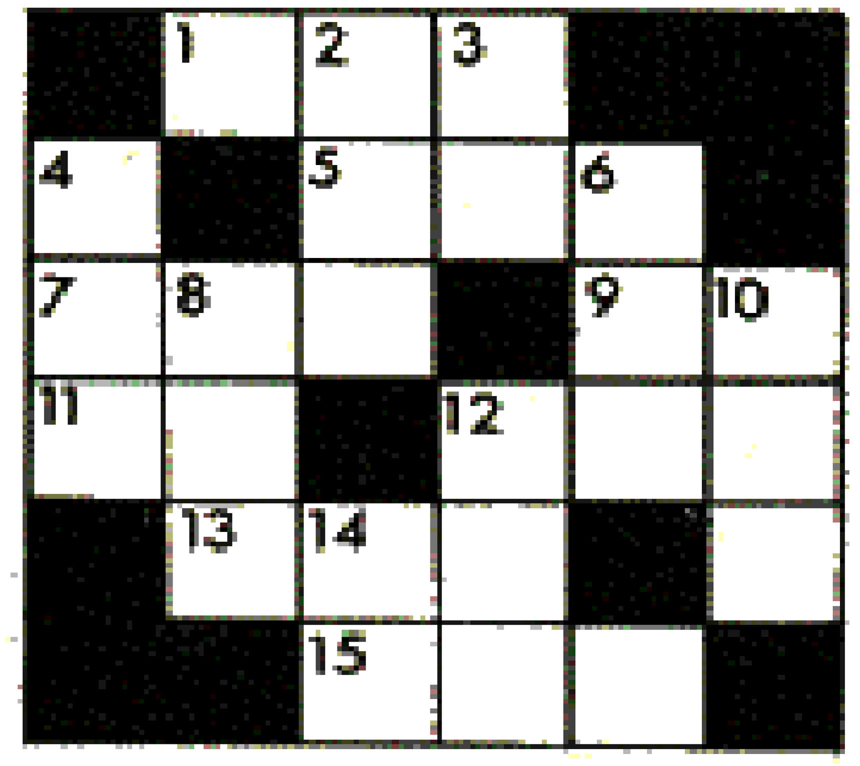
∘	//	⊥
//	//	⊥
⊥	⊥	//

TABLE 3

Dividing the numbers into positive and negative sets, members of the sets may be combined by multiplication as shown by Table 2. Table 3 shows the relation between lines parallel and perpendicular to a given line. All these tables have the same structure.

B.A.

JUNIOR CROSSFIGURE NO. 46

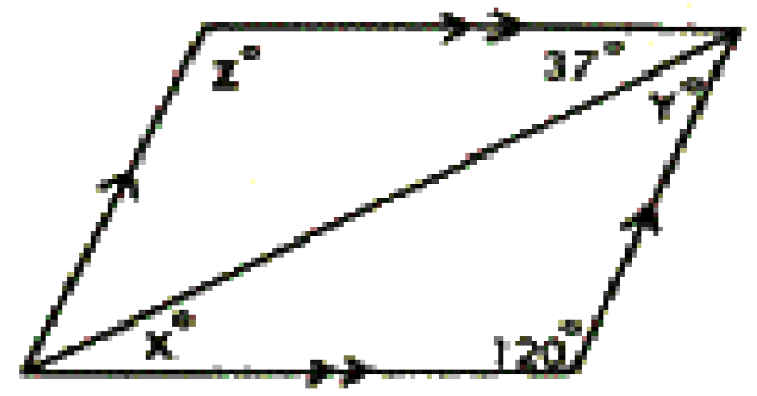


CLUES ACROSS:

1. A perfect square.
5. $(2^4)^2$.
7. Interior angle of a regular decagon.
9. Sum of the squares on all the sides of the triangle in 3 Down.
11. Angle y in the diagram.
12. Angle z in the diagram.
13. No. of kgm. in 290·4 lb. (Take 1 kgm. = 2·2 lb.)
15. Circumference in yards of a circle of diameter $227\frac{1}{2}$ yards. (Take $\pi = 3{\cdot}14$).

CLUES DOWN

2. Principal that will yield £5. 4s. simple interest in 4 months at $2\frac{1}{2}\%$ per annum.
3. Area of a triangle with vertices (0, 0); (5, 2); (3, 5).
4. 11021 ÷ 12 (base 3).
6. L.C.M. of 96 and 168.
8. $\begin{pmatrix}5\\4\\2\end{pmatrix}+\begin{pmatrix}3\\-2\\6\end{pmatrix}+\begin{pmatrix}-4\\1\\-7\end{pmatrix}=\begin{pmatrix} \\ \\ \end{pmatrix}$
10. $a^2(a-c)(2b+2c+d)$ when $a=4$, $b=3$, $c=2$, $d=9$.
12. 22 × 2 (base 3).
14. Angle x in the diagram.



D.I.B.

SOLUTIONS TO PROBLEMS IN ISSUE No. 52

SENIOR CROSS FIGURE No. 48

Clues Across: 1. 1732; 3. 27; 5. 46; 6. 208; 7. 67; 9. 401; 11. 733; 12. 810; 14. 81; 16. 750; 17. 61; 19. 54; 20. 2515.

Clues Down: 1. 14; 2. 766; 3. 200; 4. 7812; 8. 7700; 10. 1875; 13. 154; 15. 161; 18. 15.

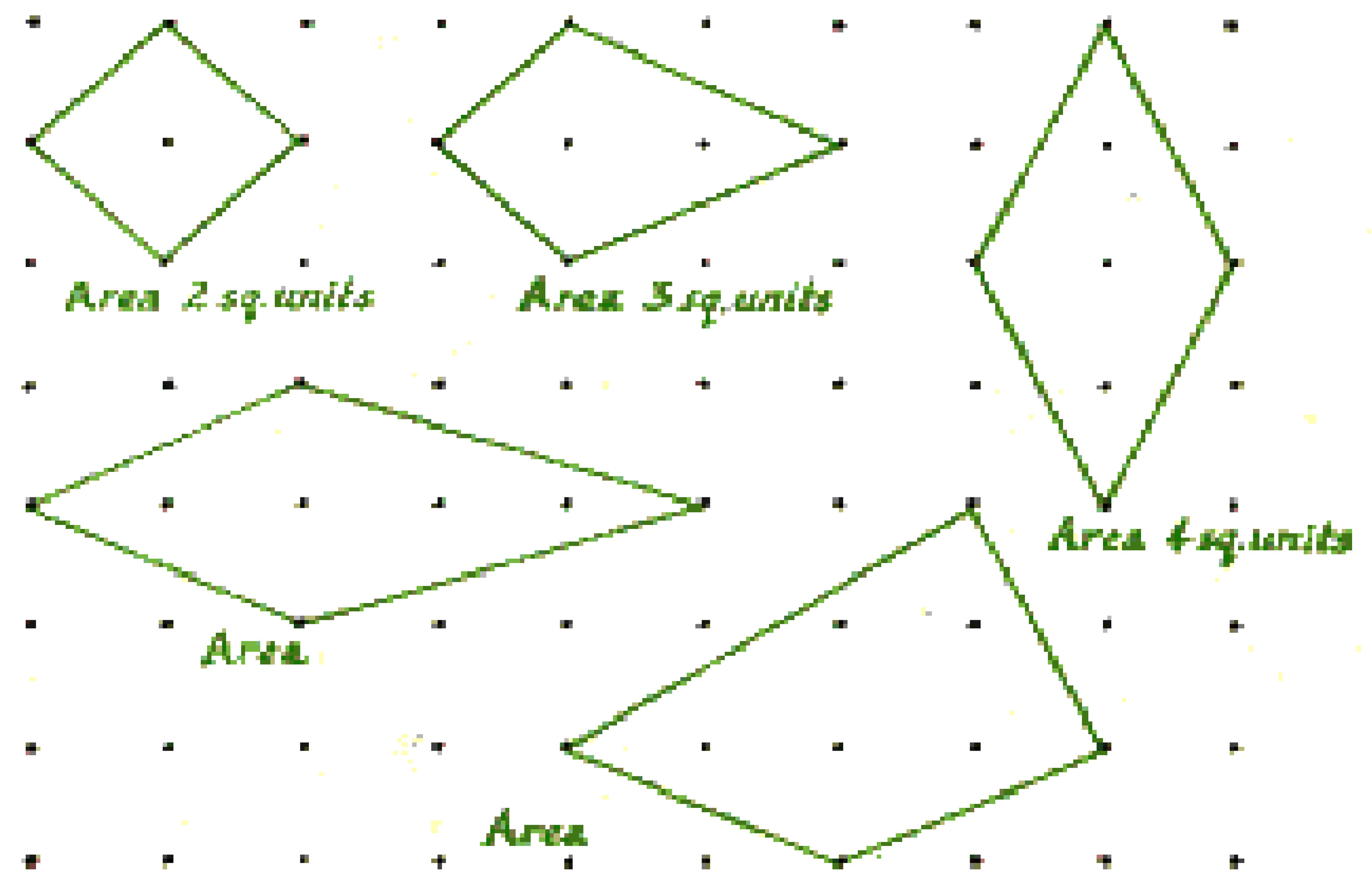
JUNIOR CROSS FIGURE No. 45

Clues Across: 2. 1728; 4. 4199; 6. 89; 7. 291; 10. 162; 12. 14; 13. 8224; 15. 1320.

Clues Down: 1. 6481; 2. 19; 3. 792; 5. 196; 8. 912; 9. 1440; 11. 282; 14. 20.

SOLUTIONS TO PROBLEMS IN ISSUE No. 53

AN AREA PROBLEM—



STITCHCRAFT—If the numbers are $(n-1)$, n, and $(n+1)$, then $(n-1)$ and $(n+1)$ are even and one is divisible by 3; hence it is divisible by 6.

SQUARE DANCE — The dancers travel towards each other at all times, hence they move a distance equal to their original separation.

SQUARE FROM A CIRCLE — The side of the square is $2\sqrt{2}$ inches and the area 8 square inches.

FOOD FOR THOUGHT—A cup of coffee, a sandwich, and a bun cost 2s. 2d.

B.A.

of Kobel, Adam Riese, Tonstal and Recorde are remembered for their contributions to this work. Tonstal, who was Bishop of London, wrote the first arithmetic book to be published in England (is he the man to blame ?) Recorde was one of the first to use the familiar equals sign =. The Germans Rudolfe and Stiffel also wrote books in which we first see the signs + and —.

One of the great events of the early part of the century was the publication by Copernicus, in 1542, of his theory of the Universe. It has been said that Columbus opened up a new world, but Copernicus opened up millions of worlds. Copernicus' work needed an improvement in the trigonometry available, he himself wrote a treatise on it, and later in the century Romanus gave a proof of the formula for Sin (A + B). Astronomers were also interested in the confusion over the movable feast days and, at the invitation of Pope Gregory XIII, solutions were suggested to this problem, resulting in the adoption in 1582, of the calendar of Clavius in which October 15th followed directly after October 4th. It was not until 1752 that the Gregorian calendar, as this was known, was adopted in England when eleven days were omitted.

In the field of practical mathematics we must mention the inventions by Leneord Digges, in 1571, of the theodolites used for measuring angles in surveying.

Finally we come to Simon Stevin, the first man to give the theory of decimal fractions. Stevin was better known among his contemporaries for his work in hydrostatics, which, since he lived in Holland, was of immediate practical importance.

A.W.B.

WHY WAS IT CALLED ?

"Volume" is a word that has acquired its present meaning by mistake. If we come across a word that we do not understand we are sometimes tempted to guess a meaning that seems to make sense. Sometimes these guesses become generally accepted, as has happened recently with "litter" and "commuter". The word "volume" originally meant a roll, particularly a roll of manuscript. The word "voluminous" was used by architects to describe ornaments with scrolls or spirals. It was sometimes used quite correctly in poetic metaphors. Milton described a serpent as "voluminous and vast". Other poets wrote of "voluminous waves" and "voluminous cloaks". Readers who did not understand the meaning of the word thought that it must mean large or bulky, and this became generally accepted. If "voluminous" means bulky then "volume" ought to mean bulk. In 1841, for the first time, an arithmetic book contained the words "the volume of a cube . . ."

"Area" to the Romans meant an open space in a city. Then the word was applied to a clear space without seats in a public building. It was a simple step from describing an area of 20 yards by 30 yards to talking of an area of 600 square yards.

In 1570, the words "area of a triangle" were used in Billingsley's translation of Euclid.

C.V.G.

CHARLIE COOK REACHES THE SIXTH

Given $x = \sin^3\theta$ and $y = \cos^3\theta$, then $x+y = \sin^3\theta + \cos^3\theta$

hence $x^{1/3} + y^{1/3} = \sin\theta + \cos\theta$

and $x^{2/3} + y^{2/3} = \sin^2\theta + \cos^2\theta$

or $x^{2/3} + y^{2/3} = 1$.

FOOD FOR THOUGHT

The bill for 4 cups of coffee, 3 sandwiches, and 7 buns is 9s. 7d., and the bill for 3 cups of coffee, 1 sandwich, and 9 buns is 8s. 4d. What would be the bill for 1 cup of coffee, 1 sandwich, and 1 bun?

C.V.G.

AN ANNUAL PROBLEM

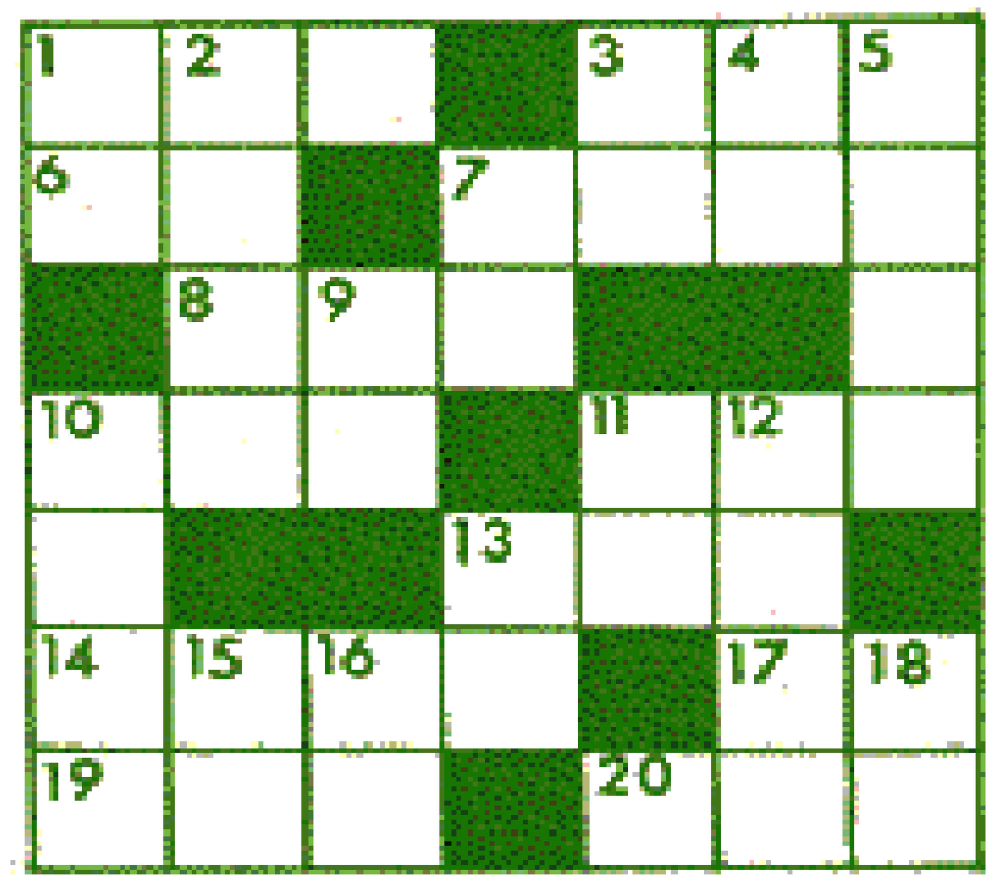
$$1967 = 19\left(99 + \frac{999}{999}\right) + \frac{666}{666} + 77 - 7 - \frac{77}{7} + 7$$

Can you find an arrangement of one 1, nine 9s, six 6s, and eight 8s which gives the value 1968 using conventional mathematical symbols only? A book token will be sent to the author of the best solution.

R.H.C.

SENIOR CROSS-FIGURE No. 49

Ignore decimal points and work to the appropriate number of figures.



Clues Down :

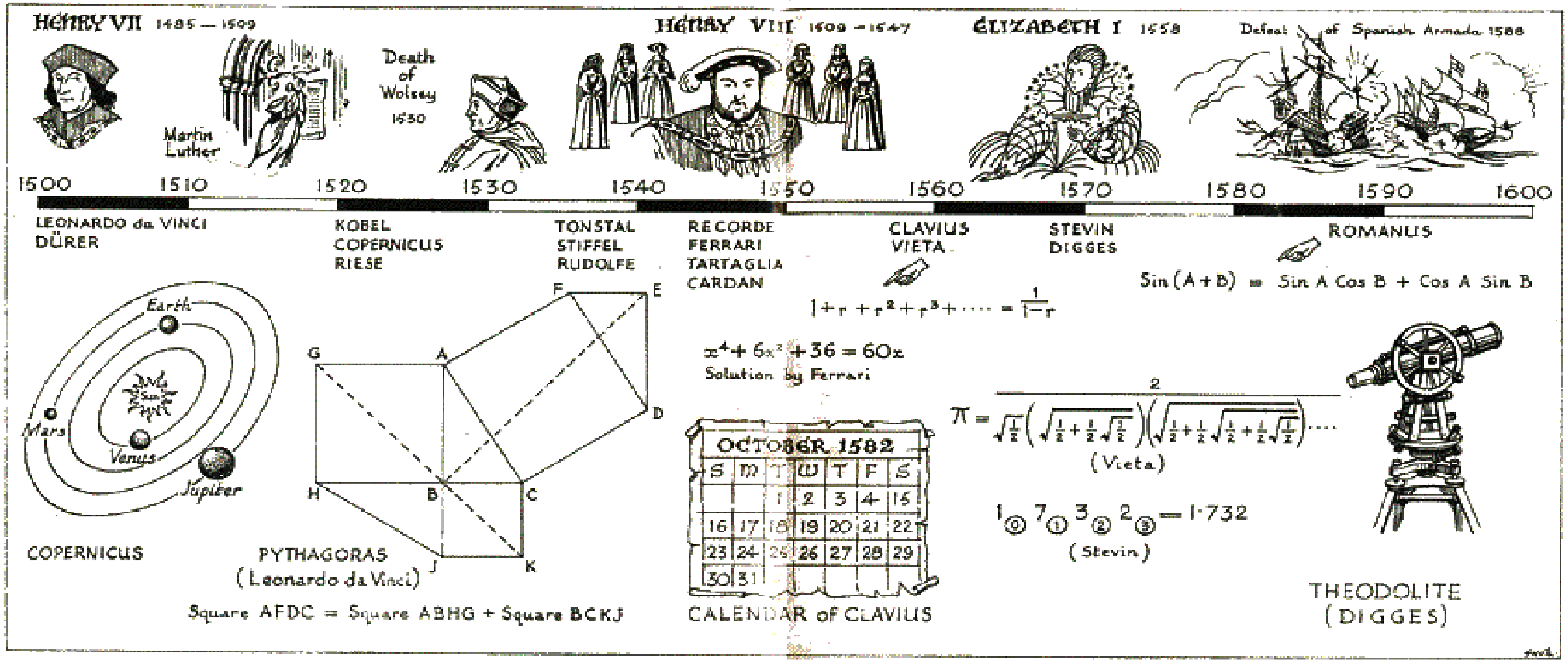
1. Area of a kite with diagonals of 4 and 6 units.
2. Rate per annum which, in 3 years 7 months, yields simple interest of £45 3s. on £144.
3. Difference between the roots of $5x^2 - 24x + 16 = 0$.
4. $\sqrt{2025}$.
5. sin 155°50′.
7. Radius of circumcircle of triangle with vertices (0,0), (2,8) and (6,3).
9. Angle to horizontal of resultant in 11 across.
10. Francs in £1 when 50.96 francs = £3 14s. 8d.
11. Find x when $x^{0\cdot5} = 3\cdot162$.
12. Minimum value of $5x^2 - 3x - 9$
13. Largest angle of triangle in 7 down.
15. The radius of a circle to which a tangent, from a point 9 in. from the centre, is of length 7·2 in.
16. $\left(\frac{1}{16}\right)^{-1\frac{1}{2}}$
18. 30 m.p.h. in ft. per sec.

Clues Across :

1. Largest volume enclosed by a surface area of 154 sq. units.
3. Total area of segments of circumcircle which lie outside the triangle in 7 down.
6. Area of kite similar to 1 down, with longer diagonal of 9 units.
7. $\frac{2}{\sin\theta}$ as a decimal when $\tan\theta = \frac{8}{15}$.
8. $4^4 - 11^2 - 1$, in base six.
10. Distance (in feet) travelled from an origin after 3 seconds if velocity is given by v = 24t ft. per sec.
11. Magnitude (in lb. wt.) of single force equivalent to forces of 8 lb. wt. at 30° to the horizontal and 12 lb. wt. at 60° to the horizontal acting on a body in the vertical plane.
13. Volume generated when $y = x^2 + 3$ is rotated about the x—axis between x = —1 and x = +1.
14. Sum to 20 terms of the series; 7, 9·72, 12·44, 15·16, . . .
17. Number of times a score of 7 can theoretically be expected when two dice are thrown 324 times.
19. The original cost of a car which is sold for £476 after depreciating by $12\frac{1}{2}$%.
20. The density (in gm. per c.c.) of a solid cylindrical rod which weighs 35·300 kg. and has a diameter of 12 cm. and a length of 30 cm.

D.I.B.

TIME CHART Nº 6A



After a long period of inactivity, the beginning of the Sixteenth Century saw Europe again taking an interest in mathematics. This was the time of the Renaissance, the re-birth of learning and scholarship; printing had just been invented giving the mathematician an opportunity of publishing his work and having it read by a wider group of people. All was set for the great advances to be made in the next few centuries.

Many of the people who helped to contribute to the mathematics of the early part of the century were also artists, one of the greatest being the famous painter, sculptor and scientist Leonardo da Vinci. His work in mathematics included a proof of Pythagoras' Theorem and in mechanics he was interested in the theory of the inclined plane and centres of gravity.

Another artist whose interest in mathematics, particularly geometry, made a lasting contribution to the subject was the German Albrecht Durer. His work was mainly connected with human proportions and the representation of solid objects on a flat canvas, the use of perspective, and he wrote several books on the subject. He is also credited with being the first person to use paper folding in the study of elementary geometry.

Pure mathematicians in the Italian Universities were concerned with the solution of equations. Scipio Ferro discovered a solution of the equation

$$x^3 + mx = n$$

and hence solved a problem that had baffled the Greeks. He did not, however, publish his solution at once and another Italian Niccolo Fontana, nicknamed Tartaglia, (the stammerer), also found the solution. Under a pledge of secrecy he told of his work to Cardan, who was professor at the University of Bologna, only to find a few years later that Cardan published it as his own work. Cardan was a man of contrast, dishonest in his dealings with Tartaglia, but, nevertheless, a serious student of mathematics who made many honest contributions to the subject, including a discussion on the number of roots an equation should have and the possibility that some of these roots may be negative or even imaginary. Ferrari, a pupil of Cardan, was the first person to solve the quartic equation

$$x^4 + 6x^2 + 36 = 60x$$

Cardan was again the publisher. The last of the great Italian mathematicians concerned with this work was Rafeal Bombelli, whose book 'Algebra' published in 1572 considered the roots of equations in great detail.

The development of algebra was not confined to the Italians, for a Frenchman, Vieta, studied with considerable success, many of the problems that had baffled the Greeks; giving, for example, the first algebraic formula for evaluating π. However, it is for his achievements in improving algebraic notation by introducing letters to represent numbers that he is most remembered.

The activities of Columbus and the other sea captains and the consequent growth in trade and travel, meant that commercial arithmetic became more important and many books were written on the subject. The names