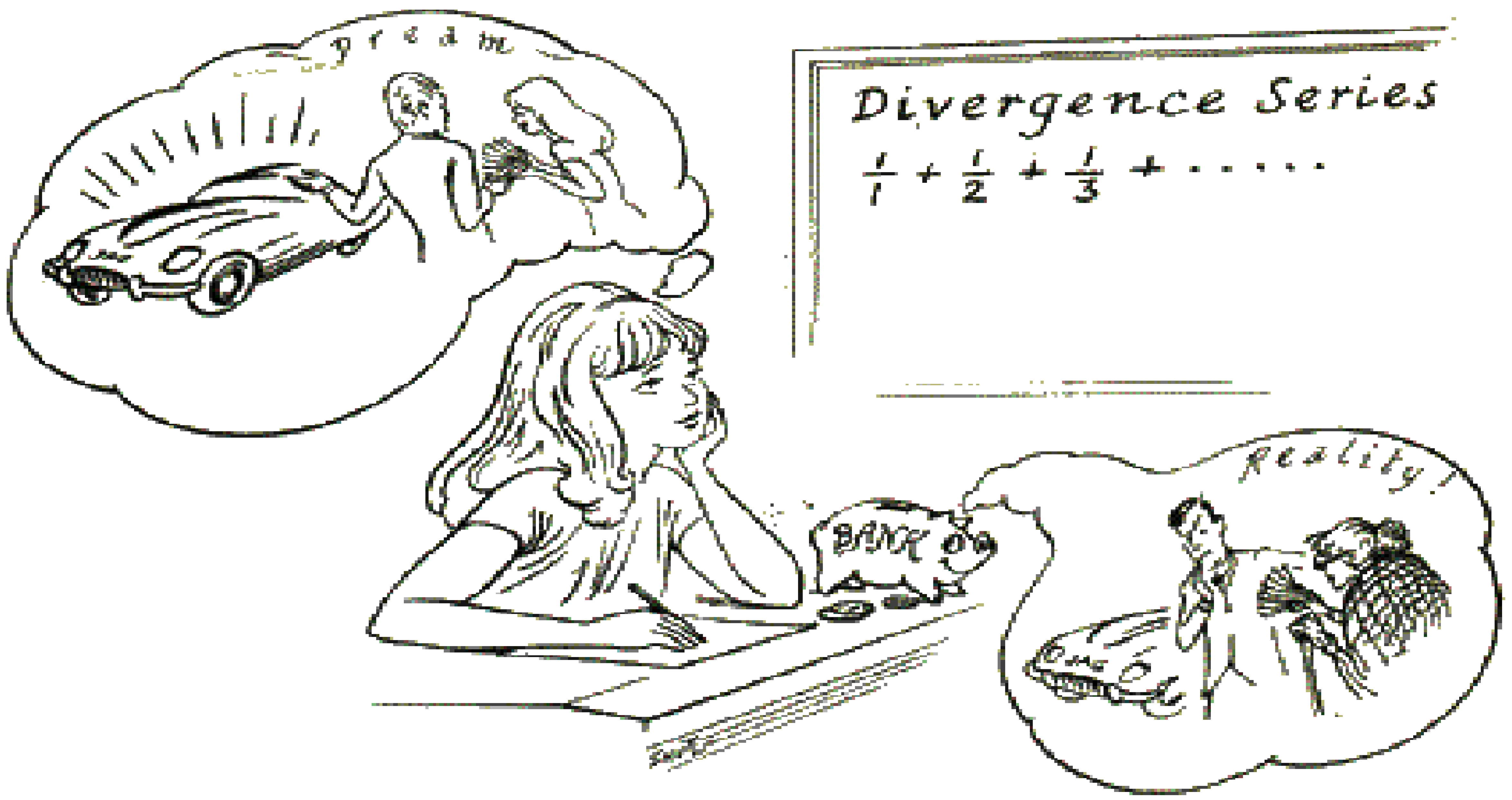
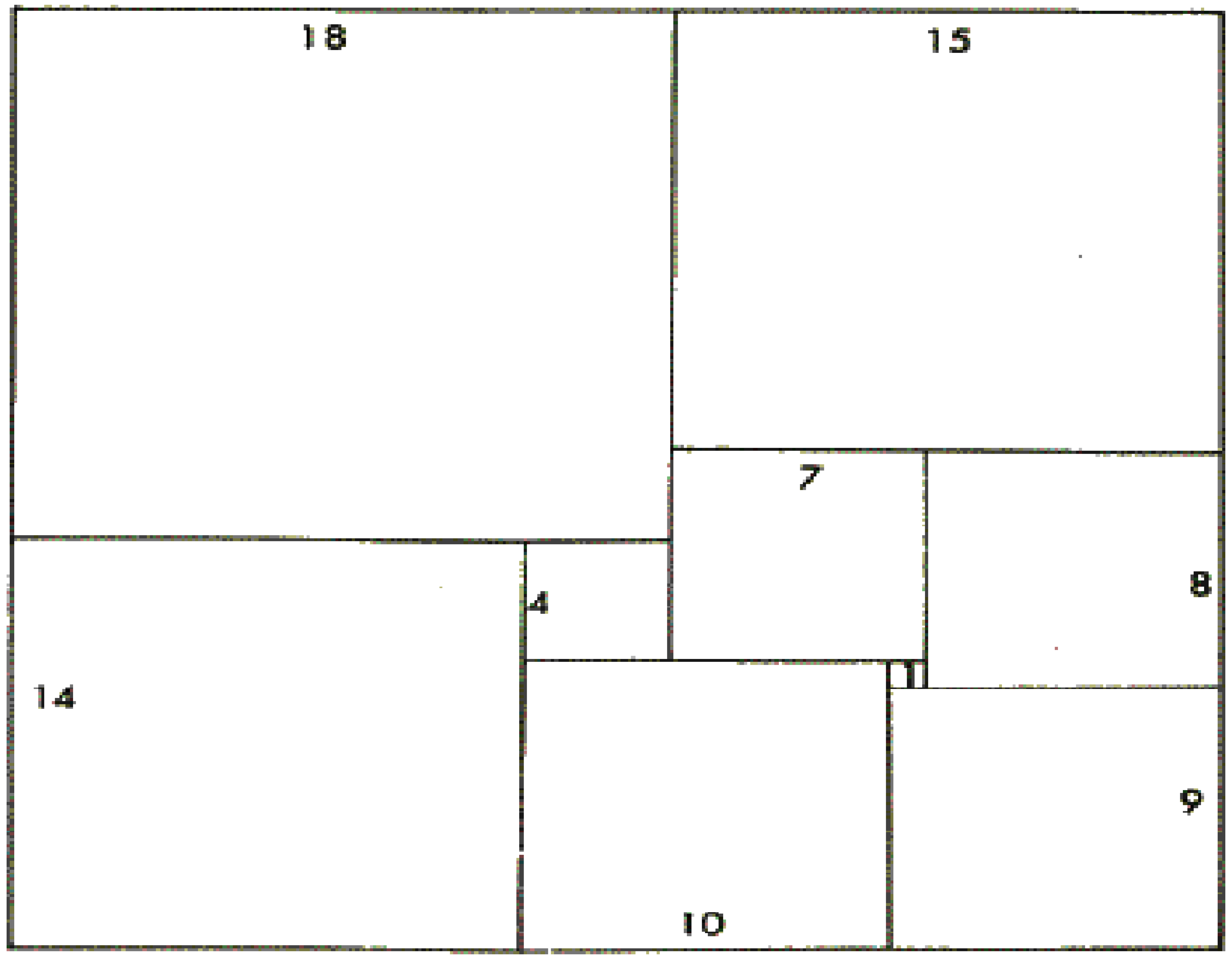
CUTIE'S DAYDREAM



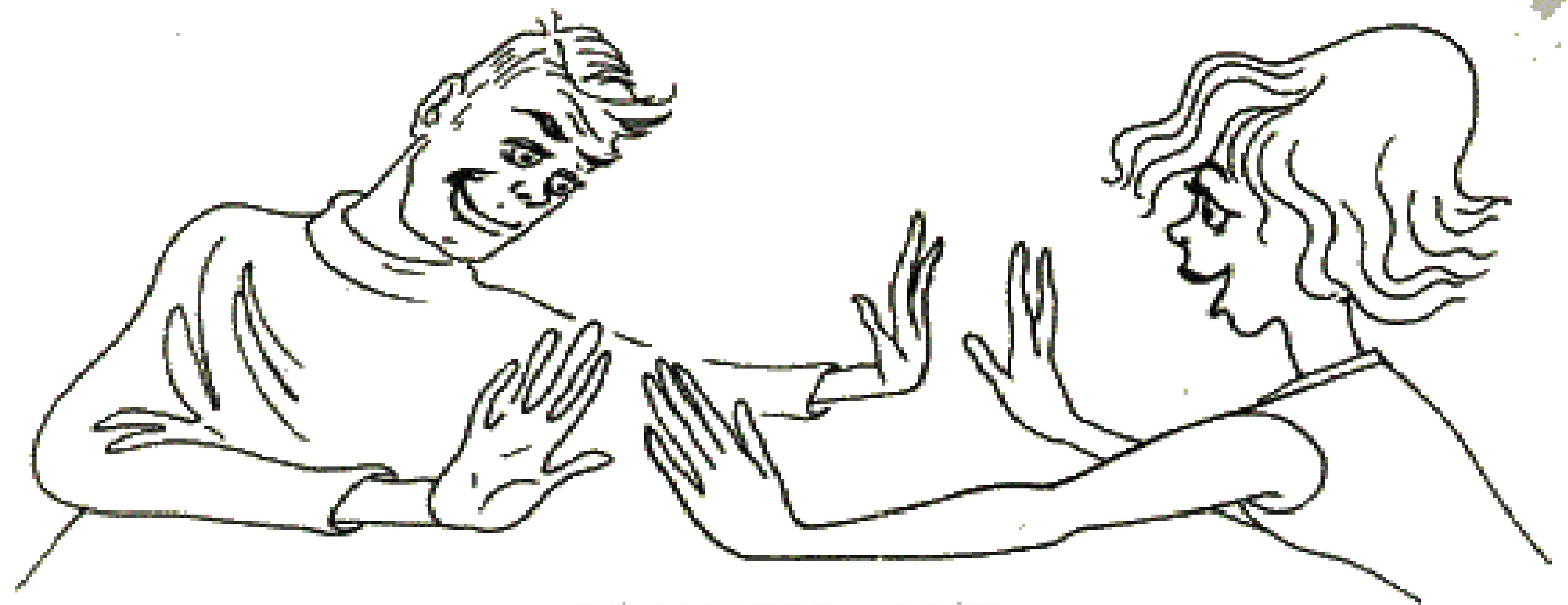
In issue No. 48, Cutie amassed a fortune by saving one penny on the first day of a month, two on the second, four on the third, doubling the amount saved each day. She found this rather difficult to keep up so the next month she saved £1 on the first day, £$\frac{1}{2}$ on the second, £$\frac{1}{3}$ on the third, and so on. She knew that this was also a divergent series and dreamed of buying a car as illustrated. Why was the piggy bank's dream so different from her own? B.A.

DISSECTION OF A RECTANGLE SOLUTION



mathematical pie

No. 56 Editorial Adress: 100, Burman Road, Shirley, Solihull, Warwickshire, England SPRING 1969



COUNTED OUT

In the kingdom of Awkwardia, they have a most peculiar way of counting. The men of that country have five fingers on the right hand but only three on the left hand. They start by counting units on the right hand and call a full hand a REE, so that 5 units make 1 Ree. They go on to count Rees on the left hand so that 3 Rees make a full hand called a LEE. 5 Lees counted on the right hand make a RIE and 3 Rie make 1 LIE and so on, alternately using the left hand and the right hand. Thus they count 1, 2, 3, 4, 10, 11, 12, 13, 14, 20, 21, 22, 23, 24, 100, 101, 102, 103, 104, 110, etc.

The King was in his counting house counting out his money. He had just finished one pile when he was called away to the telephone so he made a note of the total as 210,212 florins. Assuming that 16 Awkwardia florins make £1 sterling, how much is the pile worth in English money?

While he was away the Queen wandered in and, noticing the total, carried on counting from where the King left off. When she reached 222,222 she remembered the bread and honey she'd left in the parlour and leaving a note for the King she hurried back. Unfortunately they both forgot one important fact: the women of Awkwardia have five fingers on the left hand and three on their right, but being taught by their menfolk they count in the same order Right, Left, Right, Left, . . .

How many florins did the Queen count?

By how much will the King's interpretation of the Queen's total be wrong?

What numbers will be the same in both systems? R.M.S.

SQUARE THROUGH 4 POINTS

CAN YOU DRAW A PERFECT SQUARE HAVING ONE DOT ON EACH OF ITS 4 SIDES BUT NO SIDE IS TO TOUCH ANY OF THE WORDS WHICH ARE PRINTED HEREIN? IT IS NOT AS EASY AS IT MAY APPEAR AT FIRST GLANCE

CARTOON OF EGYPTOLOGIST

" If it's what I think it is, we have a lot of digging in front of us."

CODE OR CYPHER

A code replaces a whole word, phrase or sentence by a symbol numerical or otherwise, e.g., 21 All well, 22 Am in hospital, 23 Am broke, etc. Cyphers may be substitution, or rearrangement, i.e., a permutation. Miss M. Collingwood of the Convent of the Ladies of Mary Grammar School, Scarborough, submitted the following triple substitution cypher.

The letters A to Z are allotted the numbers 1 to 26 and the punctuation space, 27, full stop 28, comma 29, question mark 30, and inverted commas 31. Each number is then converted into a five digit binary number, e.g., 9 to 01001 and 30 to 11110. Every 0 is now replaced by any word with an odd number of letters and each 1 by any word with an even number of letters.

Here is a message: " Can you understand this code, which we will be decoding? " Counting the letters in each word gives odd, odd, even, even, even/odd, even, even, even even, i.e., 00111 01111 which is denary in 7 15 and in letters G O. The message is GO.

Can you decode the following message :—

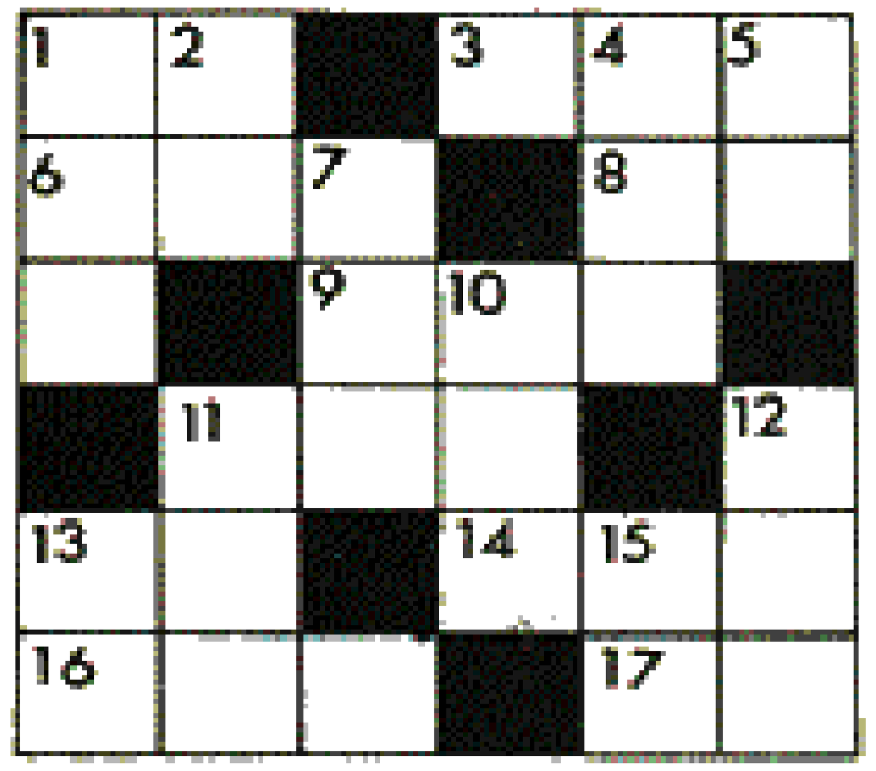
"Listed goods fit your requirements. The cover will match well, all in blue waterproof plastic. All goods have our guarantee. We will allow your firm a discount for goods paid in cash. All our experts will be ready for consultation when you require advice. Our price competitive. Yours faithfully, . . . "

A CALL SIGN FOR THE POLICE

Multiply 37037037 by 27.

By what number should 37037037 be multiplied to get the same result upside down? R.H.C.

JUNIOR CROSS FIGURE No. 49



CLUES ACROSS

1. Perimeter of quadrilateral with vertices of (12,0), (24,5), (12,10) and (0,5).
3. Angle ABC in a parallelogram ABCD when angle BAD=44°.
6. Find x^2 if $(x+12)(x-12)=297$.
8. Reflection of (0,4) in the straight line which passes through (1,1) and (3,5).
9. 4021 ÷ 12 (base-five).
11. Express 10201 (base-three) in the base-ten system.
13. $(4,5)+(-7,2)+(6,-3)$.
14. $7.28^2-2.72^2$.
16. Total number of diagonal struts on all regular polygons from the six-sided to the sixteen-sided polygon.
17. 28 per cent of £1.25 in New Pence.

CLUES DOWN

1. Sum (in degrees) of the interior angles of a pentagon.
2. Longer diagonal of 1-across.
4. $5\sqrt{7^{15}}$.
5. One of the equal base angles of an isosceles triangle in which an exterior angle at the vertex is 124°.
7. Area of quadrilateral in 1-across.
10. 23×13 (base-five).
11. Area (to nearest sq. cm.) between concentric circles of radii 3.5 cm. and 7.6 cm., respectively. ($\pi=3\frac{1}{7}$).
12. 2s. 9d. as a decimal of 16s. 8d.
13. Value of x when $2x+3=10$.
15. Angle (to nearest degree) between solution vector and horizontal in 13-across.

D.I.B.

GOING UP?

The store manager was obviously very proud of his newly-installed 30 foot long ascending escalator. He explained that its normal speed is half his walking speed and that he takes $3\frac{1}{5}$ seconds longer to walk steadily up and immediately down again (against the direction of movement) than when it is travelling at half speed.

What is the escalator's normal speed? D.I.B.

What is the connection between 4472 and the Scottish Express problem? *See page 440* B.A.

Find the value of $2^5.9^2$.

What other four digit number has the same peculiarity? R.H.C.

SOLUTIONS TO PROBLEMS IN ISSUE No. 55

THE AUSTRIAN PRETZEL DISSECTION—11 pieces by cutting vertically.

REVERSIBLE SQUARES—There are infinitely many numbers, e.g., $103^2=10609$, $301^2=90601$.

DO NOT FORGET—An elephant.

FOR LEFT-HANDED NUT CRACKERS
Across: 1. 001; 3. 01; 5. 1011; 7. 01; 8. 011; 9. 0011; 11. 11; 12. 111.
Down: 1. 01; 2. 11; 3. 0111; 4. 101; 6. 0101; 7. 011; 8. 001; 10. 11.
"The clues and answers used numbers in the binary scale written backwards".

SENIOR CROSS FIGURE No. 51
Across: 1. 2211; 3. 76; 5. 58; 6. 122; 7. 64; 9. 308; 11. 228; 12. 225; 14. 64; 16. 600; 17. 63; 19. 54; 20. 1678.
Down: 1. 25; 2. 286; 3. 720; 4. 6280; 6. 1386; 8. 4250; 10. 5265; 13. 204; 15. 467; 18. 38.

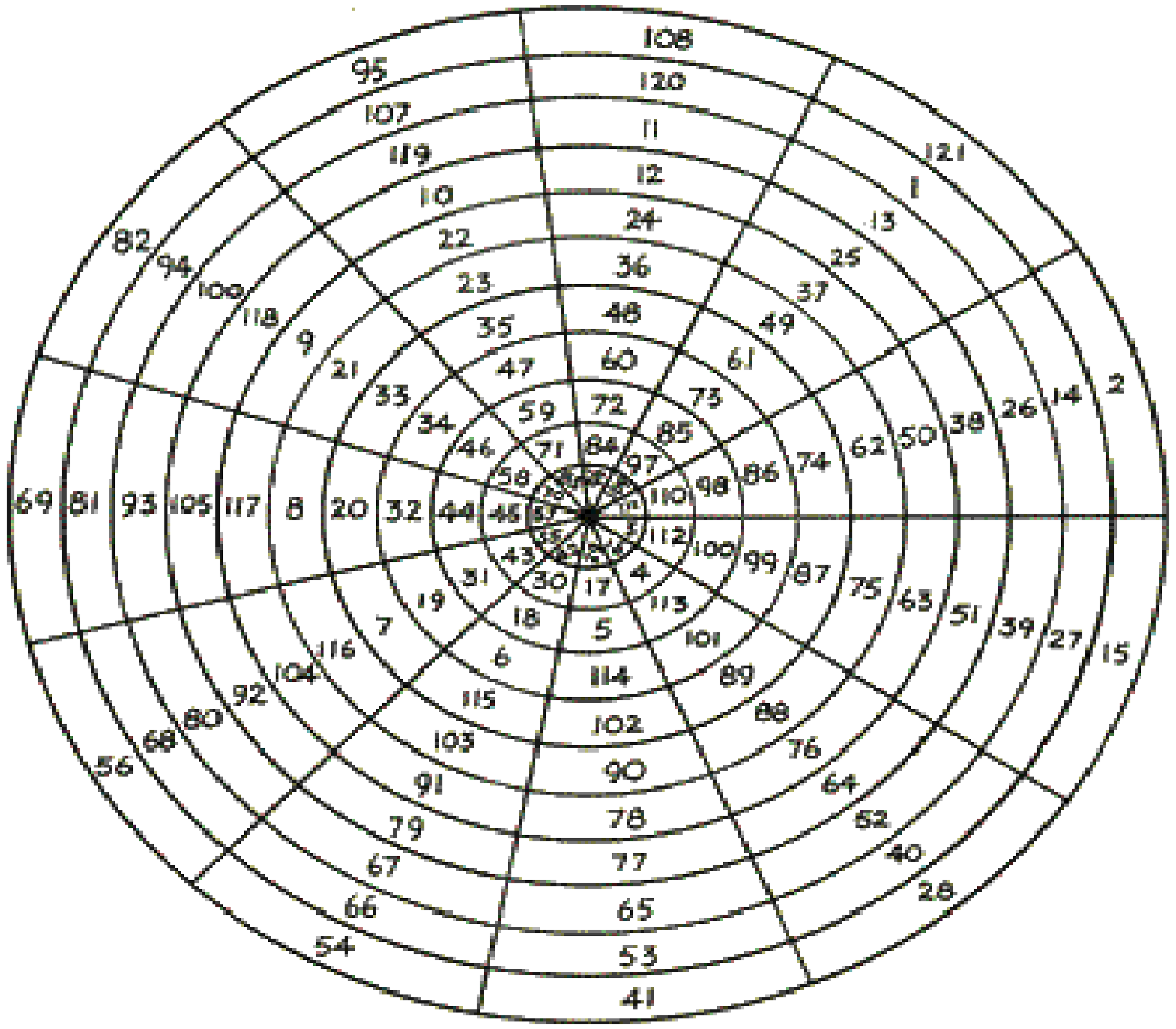
JUNIOR CROSS FIGURE No. 48
Across: 1. 169; 3. 13; 5. 51; 6. 135; 7. 212; 10. 231; 12. 198; 14. 28; 16. 26; 17. 254.
Down: 1. 15; 2. 612; 3. 13; 4. 352; 6. 123; 8. 128; 9. 521; 11. 125; 13. 96; 15. 84.
Clue 9 down should have read $\log_9 x = 9$. B.A.

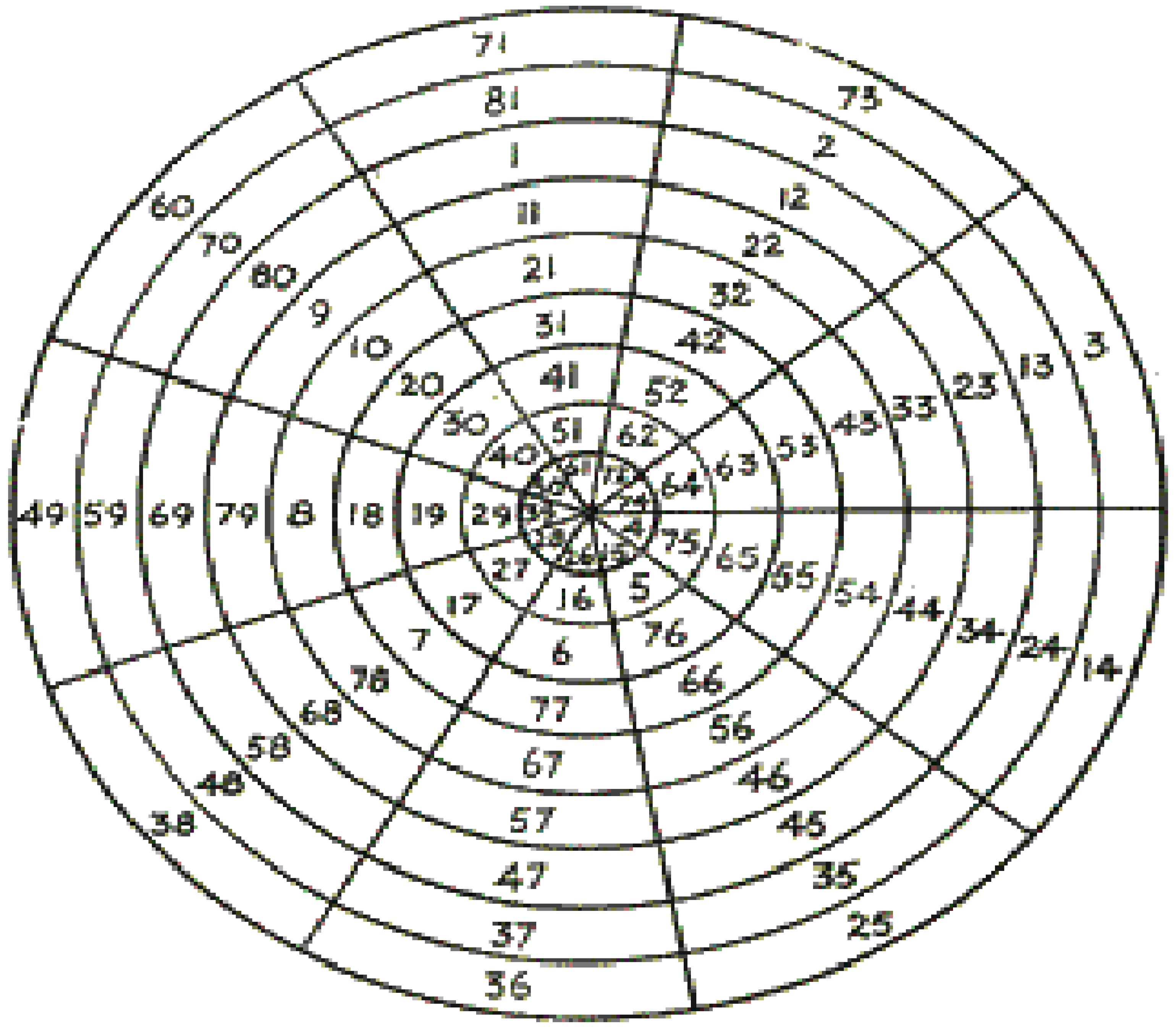
Continued from page 441

The "shell" of the complete model takes the form of a hemi-ellipsoid while the "head" has that of a hemisphere.

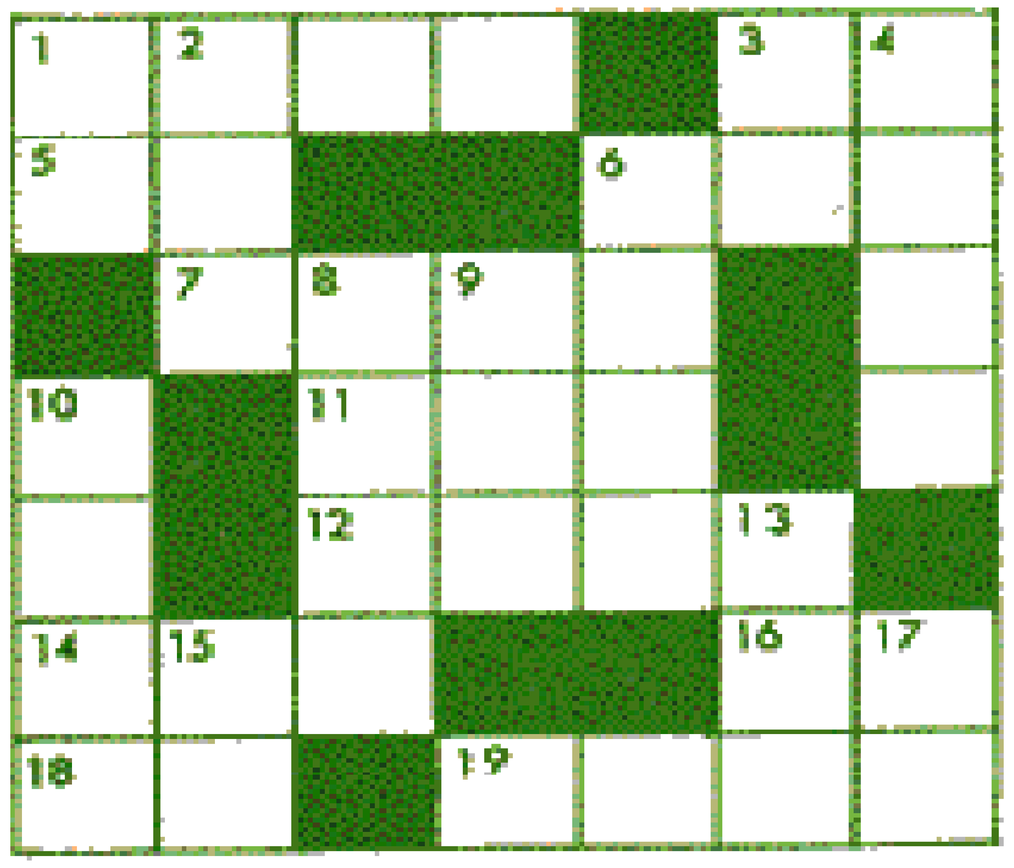
Readers are invited to send functions which produce interesting shapes to the Editor. Book tokens will be sent to those whose entries are published in Pie. D.I.B.

MAGIC CIRCLES SOLUTION





SENIOR CROSS FIGURE No. 52



Ignore signs and decimal points in solutions, and work to appropriate number of significant figures.

Clues Down

1. $y \propto x^2$ and $y=16$ when $x=2$. Find y when $x=3$.
2. a, b and c (smallest whole numbers, in order) for $ax^2+bx+c=0$, if roots are $\frac{3}{4}$ and -2.
3. Number of faces of regular polyhedron with 20 vertices and 30 edges.
4. $\text{Log}_{10}4$ given that $\log_{10}2=0.3010$.
6. Sum (in degrees) of interior angles of an octahedron.
8. Shortest surface distance (to nearest 10 miles) between 30°N 115°W and 30°N 45°E. Radius of earth =3960 miles.
9. Gradient at $(-2, -10\frac{1}{4})$ on $y=x^3+2x^2+\frac{21x.}{4}$
10. 1 hectare (10,000 sq. metres) in acres, given that 1 metre=39.37 in.
13. Maximum s when $s=128t-16t^2$.
15. 6 tan.234°.
17. The distance of a point from the centre of a circle of radius 25 cm., if the tangent from the point is 60 cm.

Clues Across

1. Sum of whole numbers, which are not multiples of three, between 1 and 101.
3. $\left(\frac{1}{2-2}\right)^2$
5. Hypotenuse $(6x-7)$, if other sides of right-angled triangle are $(2x+1)$ and $5x$.
6. Total number of arrangements of MATHS.
7. $\text{Log}_{10}5$, from information in 4-down.
11. The present value (in £) of goods which, after two years, will be worth £404 5s. 0d., having depreciated by $12\frac{1}{2}$ per cent each year.
12. 233002 (base-five) in base-ten.
14. Radius of circle if chord of length 14.4 cm. is 3.0 cm. from centre.
16. Volume of pyramid of height 14 cm. with a triangular base of sides 5 cm., 5 cm. and 6 cm.
18. True speed of ship sailing at 15 knots on a bearing 060° in a current of 6 knots on a bearing 300°.
19. $\{y=(4x-7)(x-3)\} \cap \{y=17x-44\}$

Smaller solution first.

D.I.B.

SERENDIPITY

To test our electronic calculating machine we try to fill up all available stores with sevens (111 in binary). To do this we set up a certain 15 figure number N (9 figures before and 6 after the decimal point) and multiply it by a certain 2 digit number D. The result is 7,777,777,777.777768. If I tell you that there is something regular about the appearance of N and that D is greater than 60, you should be able to find them. It may help to know that there is something odd about $\sqrt{N}$ as well.

R.M.S.

"I wish I had a computer to help me choose my clothes."

CALENDAR 1969

+ JUN	+ SUN	+ 1 8 15 22 29*
	+ MON	
+ SEP DEC	+ TUE	+ 2 9 16 23 30*
	+ WED	
+ APR JUL	+ THU	+ 3 10 17 24 31*
	+ FRI	
+ JAN OCT	+ SAT	+ 4 11 18 25
	+ SUN	
+ MAY	+ MON	+ 5 12 19 26
	+ TUE	
+ AUG	+ WED	+ 6 13 20 27
	+ THU	
+ FEB MAR NOV	+ FRI	+ 7 14 21 28

* *Thirty days have September —*

To use the nomogram, lay a ruler across the calendar from the month to the number of the day of the month. The cross in the centre column that lies under the ruler is adjacent to the day of the week on which it falls.

C.V.G.

SCOTTISH EXPRESS

Two trains start at the same time, one from London for Edinburgh and the other from Edinburgh for London. If they arrive at their destinations one hour and four hours respectively after passing one another, how much faster is one train than the other.

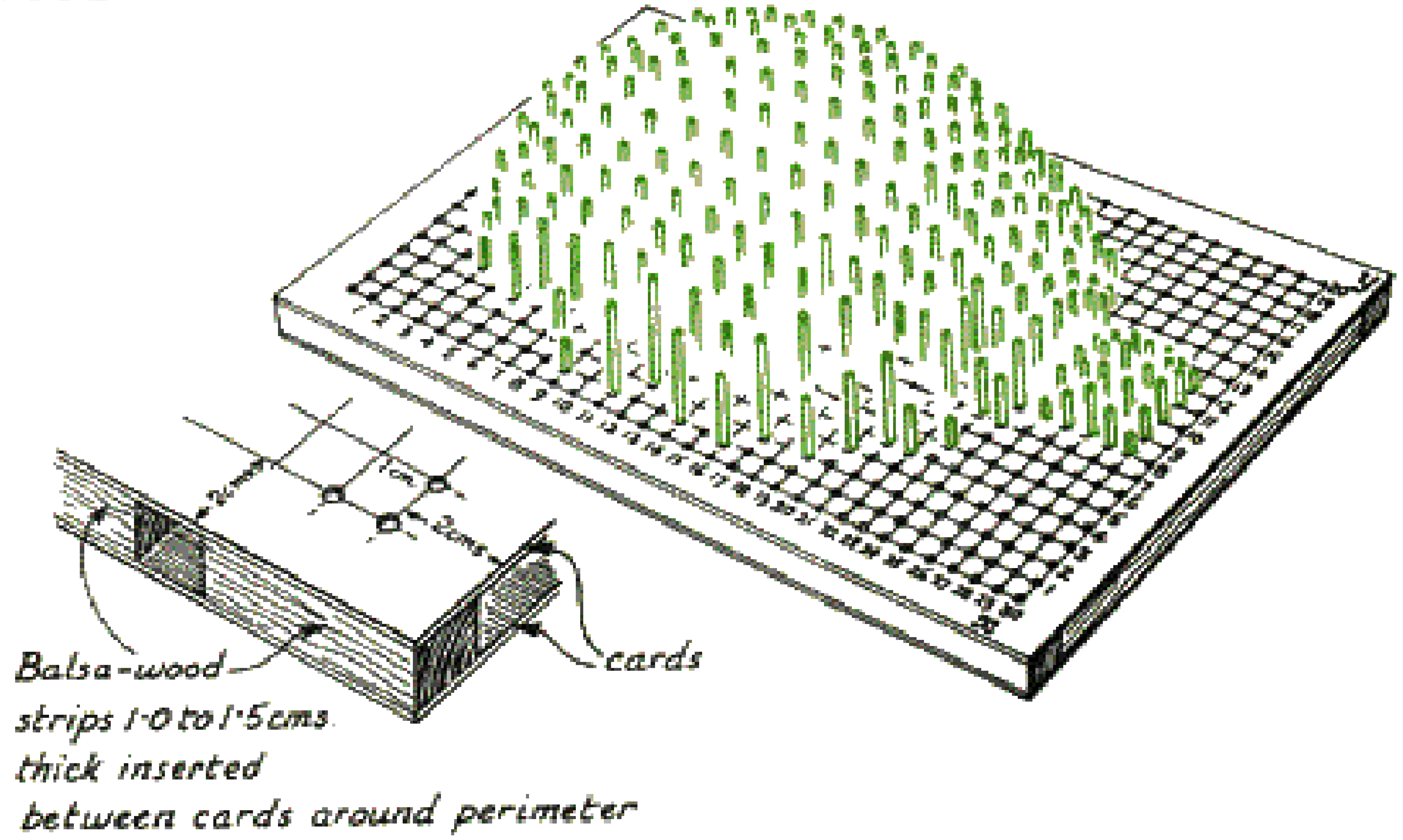
R.H.C.

TEDIOUS ?

If $3(230+e)^2 = 492t04$, find t by simple arithmetical arguments.

R.H.C.

Model Functions



While functions of one variable may be pictorially represented on graph paper, functions of two variables may be effectively shown as three-dimensional models.

Plotted on the same set of axes (x, y and z), the following two functions produce a shape which has some resemblance to that of a tortoise.

(i) $z^2 = 100 - (\frac{4}{5}x - 10)^2 - (y - 10)^2$;
(ii) $z^2 = 9 - (x - 27)^2 - (y - 10)^2$.

Cut two rectangular pieces of card 34 cm. by 24 cm. and on one construct a lattice of 1 cm. squares (30 by 20 cm.) allowing for a 2 cm. border around the perimeter. With a meat skewer, or pointed instrument of similar thickness, pierce a hole at each vertex of the 1 cm. squares. With balsa-wood strips inserted between them around the perimeter, the two pieces of card should be mounted so that they are equally spaced (between 1 cm. and 1.5 cm.).

Label the axes so that $0 \leqslant x \leqslant 30$ and $0 \leqslant y \leqslant 20$, 1 cm. representing one unit. With the same scale, the vertical (z) axis is given by the straw length.

A table of values need only consider alternate whole number values of x and y for function (i). Within the range $0 \leqslant x \leqslant 25$ and $z \leqslant 0$, this table provides an adequate set of points, if continued to $x = 25$:—

x	0	1	1	1	2	2	2	2	2	2	3
y	10	8	10	12	5	7	9	11	13	15	4
z (to 1 d.p.)	0	3.4	3.9	3.4	2.1	4.5	5.3	5.3	4.5	2.1	2.5

For function (ii), consider all positive values of z given by $26 \leqslant x \leqslant 30$.

As straws are pushed through the appropriate holes in the top card and supported by the base card below, the distance between the two cards must be added to the z value for the straw length. For example, a distance apart of 1.5 cm. would necessitate a straw length of 3.4+1.5 cm. when $x=1$ and $y=8$. A quick-drying adhesive cement is recommended to keep the straws in position.

Continued on page 442