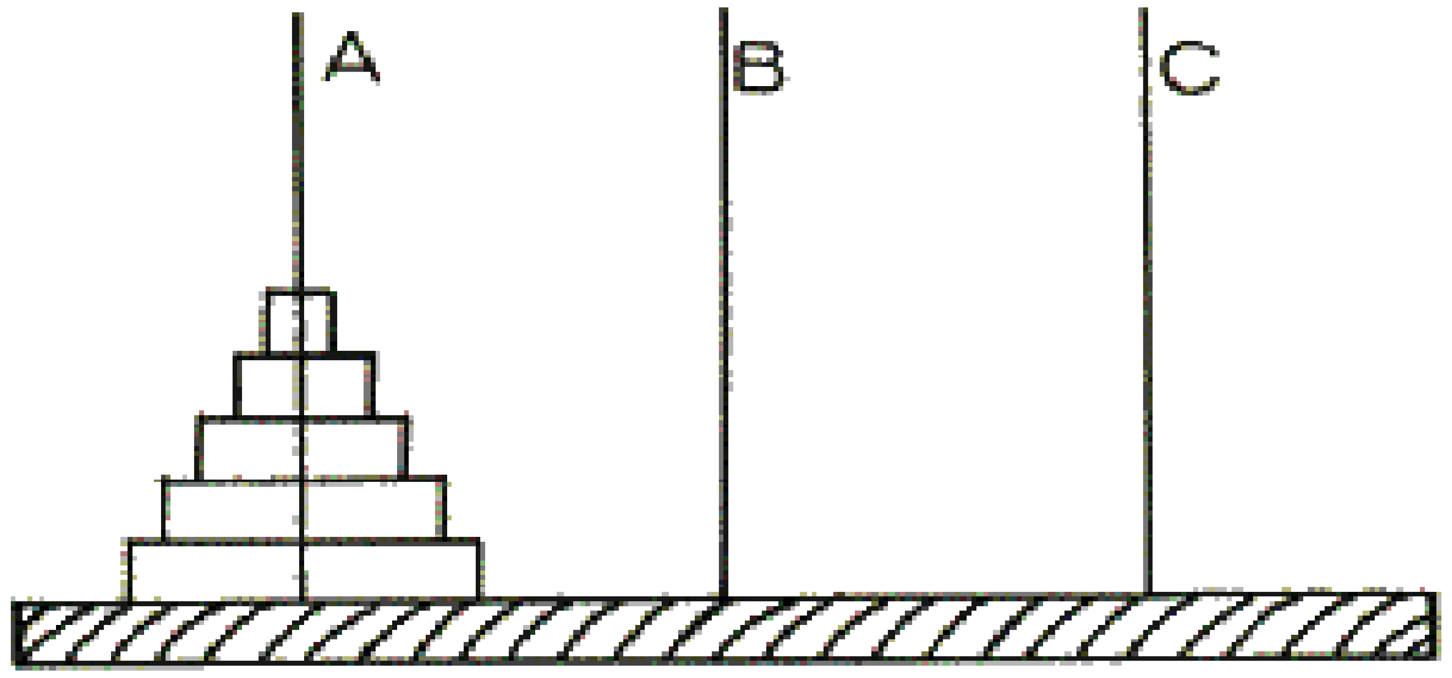
THE TOWER OF HANOI



Many of you have probably seen at some time the puzzle called the "Tower of Hanoi." The illustration shows the side view of five blocks which fit on to the spikes A,B,C. The aim of the game is to move the tower from spike A to spike C, moving only one block at a time from one spike to another. Easy?—yes, but putting a large block on top of a smaller one is not allowed. If you find difficulty now, try reducing the number of blocks from five to four, three, two and look for a system. The least number of moves for five blocks is thirty-one. Can you explain why? Find out how many times each block moves. Mr. Whittell of the Harold Malley School, Solihull, suggests as further investigation: how far does each block move horizontally if AB and BC are each one unit?

How far does each block move if the tower is moved from spike A to spike B?

E.G.

LETTUCE COS?

The hypotenuse of a 3–4–5 right-angled triangle is trisected. The vertex containing the right angle is joined to the point of trisection nearer the 3-unit side. How long is this line?

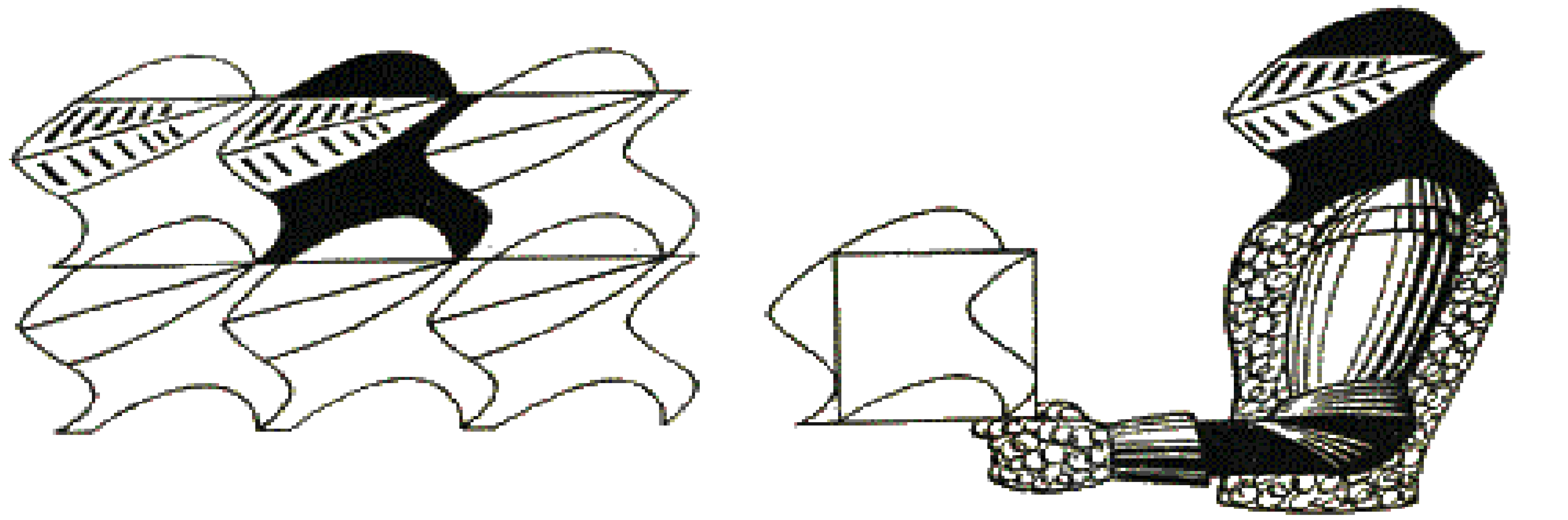
Repeat the calculation with a 5–12–13 right-angled triangle. Generalise the result with a right-angled triangle with sides a, b, c units where $a>b>c$.

B.A.

FOOTSTEPS OF TIME

A point P moves in such a way that its x-co-ordinate is $\frac{1-t^2}{1+t^2}$ and its y-co-ordinate is $\frac{2t}{1+t^2}$, where t is the time in seconds. Draw the path the point P traces out. At what time will P arrive at the point $(-1,0)$?

E.G.



Merlin's design for helmets of the Knights of the Round Table. (easy stacking!)

mathematical pie

No. 59 *Editorial Address:* 100, Burman Road, Shirley, Solihull, Warwickshire, England **SPRING, 1970**

LOOK BEHIND THE MIRROR No. 2

In issue No. 57 (Summer 1969), we play a game with Alice in Numberland behind the Looking Glass where the base of the numbers was -10 and not 10. In consequence Alice counted thus, 1,2,3,4,5,6,7,8,9,190 $(100-90+0)$, 191 $(100-90+1)$, 192, 193, ... 180 (means 20 in our numerals normally), ... 170, . . . 290 (100 in our numerals), . . . 390 . . . (Note that 10 in base minus ten means -10 and 326 means $3(-10)^2+2(-10)+6$.

Now start to complete the addition table in base minus ten. $1+2=3$, all in base minus ten, . . . $6+5=191$ all in base minus ten which is eleven in our numerals, so we begin to get

Base −10	0	1	2	3	4 9	190	191 . . . . 180 . . . .
0							
1							
2							
.							
.							
190	190					199	170
191	191						
.							
.							
196						181	

This table brings to your notice $180+190=170$ base minus ten.

Base minus ten	Base ten	Base minus ten	Base ten
180 +	20 +	160 +	40 +
190	10	130	70
270	30	290	110

We can now generalise the addition technique

Base minus ten	Base ten	
736 +	700 − 30 + 6	676 +
475	400 − 70 + 3	335
19191		1011
cba		

Column (a) $6+5=11$, put down 1 and carry -1
(b) $3+7=10$, and carry -1 equals 9
(c) $4+7=11$, put down 1 and carry -1. As there is nothing from which to take the one, put down 19 in front of the answer obtained so far. This is always the procedure when there is no number in the column to which 1 has to be carried. Now try to work out this one and check your answer by conversion to base ten. In base minus ten $693+548$.

R.H.C.

LOTTERY

From a drum containing fifty tickets numbered consecutively, two tickets are drawn without replacement. What is the probability that the difference of the numbers on the two tickets is 10 or less?

ROUND TRIP

Draw a quadrilateral and a pentagon, put in all the diagonals. In each figure try to trace out all the lines without lifting the pencil and without following the same line more than once. After testing other polygons similarly, can you discover a rule which determines the figures which can be universally drawn and can you explain it?

D.I.B.

GREENGROCERY

In the High Street of our town, there are three greengrocers. Oranges were on sale at the same price in each shop. Last Tuesday, shop A sold 21 oranges, shop B sold 29 oranges and shop C sold 37 oranges and yet each shopkeeper took the same amount. HOW?

R.M.S.

ANTI-SOCIAL

If the hands of a clock are both moving in an anti-clockwise direction, estimate the real time when the hands are exactly together between eight and nine assuming that they show the correct time at midday. What is the exact time?

What will the hands be showing when the hands would normally have been together between eight and nine?

If the minute hand only moves in an anti-clockwise direction and the hour hand moves in the clockwise direction, estimate the real time when the hands are exactly together between three and four. Find the exact time.

What will the hands of the clock be showing when the hands would normally have been together between three and four?

R.H.C.

MISSING FIGURES

Can you reconstruct the multiplication sum shown on the right?

If you concentrate on the figures of the second column, they will restrict the possibilities to two.

```
     3 4 5 ×
       * *
   ---------
     * * * *
   * * * *
   ---------
   * * 7 6 *
```

R.H.C.

THE NEW PAPER SIZES

Many of you will have heard strange rumours of paper sizes such as A_4 instead of the more familiar quarto or foolscap. It is however a completely rational system and you may be interested in the way in which they are derived. The basic sheet is A_0 which has an area of one square metre and its sides are in the ratio $\sqrt{2}:1$. If we let the width be x cm., the length will be $\sqrt{2}x$ cm. and its area will be $\sqrt{2}x^2$. Hence $\sqrt{2}x^2 = 10^4$ so $x^2 = 10^4/\sqrt{2} = 7070$, and x is 84.09. Thus the size of a sheet of paper is 84.09 cm. by 118.9 cm.

Each of the sizes A_1, A_2, A_3, A_4 etc., is obtained by folding and halving the longer side.

R.M.S.

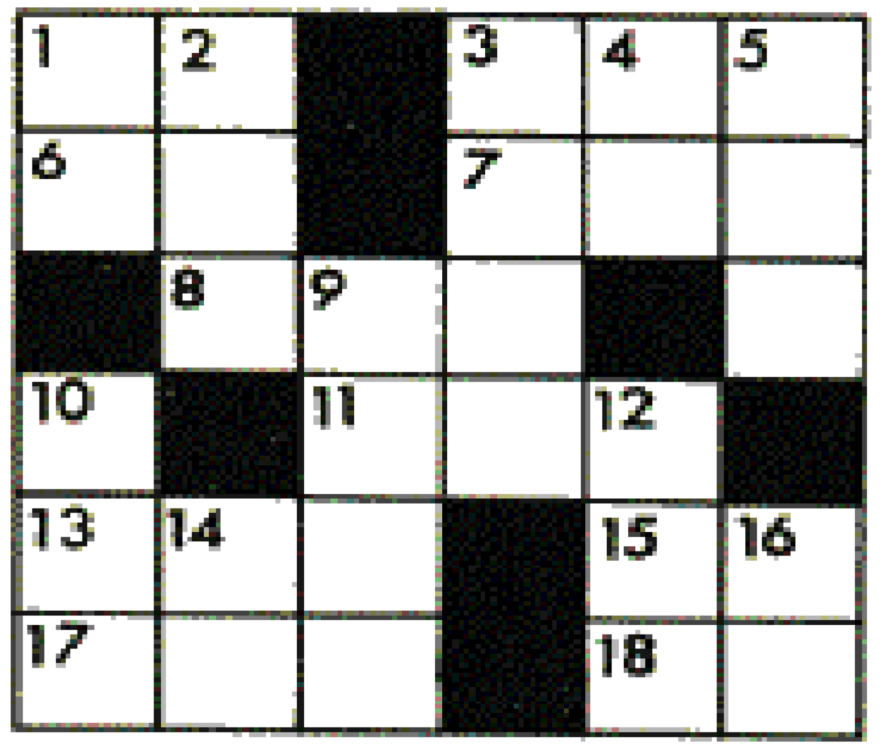
ASK DAD

Is a reasonable way of defining the width of a lawn mower as n miles per acre? What could this mean?

How will going metric affect such a unit?

B.A.

JUNIOR CROSS FIGURE No. 51



CLUES ACROSS:

1. n { A∪B∪C } if A = { 2,4,6,8,10, 12 },
   B = { 1,3,5,7,9,11 } and
   C = { 3,6,9,12 }.
3. B′∩C in 1-across, written in ascending order.
6. Reflection of (2,5) in line y = x.
7. Cost of 1 kg if 8.500 kg cost £42.75½.
8. 2122 ÷ 7 in base-eight.
11. 6.0849 correct to three significant figures.
13. Position of (7,5) when rotated 90° clockwise in the plane about the point (11,2).
15. $\sqrt{(4)^3} \times \sqrt[3]{(27)^2} - 5°$.
17. Diagonal divided by side in any regular pentagon.
18. $3\underline{a} - 2\underline{b}$ given that $\underline{a} = (7,3)$ and $\underline{b} = (8,2)$.

CLUES DOWN:

1. Number of sides forming a convex polygon which has an interior angle-sum of 2340°.
2. Bearing of A from B if bearing of B from A is 042°.
3. 254 × 24 in base-eight.
4. Value of x if $\frac{2x}{5} + 3 = 7$.
5. $\begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$ if $\begin{pmatrix} 2 & -3 \\ 5 & -8 \end{pmatrix} \begin{pmatrix} 5.5 \\ 3.0 \end{pmatrix} = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$
9. Our year 1970 in Octavia!
10. 1100 − 101 base-two.
12. Value of x if $\frac{2.5}{x} = \frac{20}{7}$.
14. Base of a triangle with a perpendicular height of 4 units and an area of 9.2 sq. units.
16. 54 km per hr. written in m per sec.

D.I.B.

SOLUTIONS TO PROBLEMS IN ISSUE No. 58

THE CASE OF ASS—By two applications of the theorem of Apollonius, XZ is $7\sqrt{2}$ cm.

SENIOR CROSS FIGURE No. 54
Clues Across: 1. 102; 3. 133; 5. 447; 7. 27; 9. 154; 10. 75; 11. 45; 12. 39; 14. 41; 16. 166; 18. 445; 19. 116.
Clues Down: 1. 112; 2. 24; 3. 17; 4. 329; 6. 42; 8. 779; 9. 19; 12. 384; 13. 56; 15. 156; 16. 15; 17. 61.

THIS TAKES THE CAKE!—The editor thanks all contributors for their interpretation of Alice's remarks. It was easy to show that each letter represented a reflection about the side labelled by the letter and that " Meet a team at tea " returned the cake to its original position. B. Marshall of Dollar Academy also showed that the operation was associative and was the most detailed of the contributions. A book token has been sent to him.

HICCUPS—The general strategy to bring all three cups upright in exactly n moves is to invert one the right way up and one upside down for the first (n − 1) moves and then invert the two that are the wrong way on the next move.

MAGIC TRIANGLE
There is no arrangement which does not include 5 at a vertex.
One possibility is—

```
      9
     4 3
    2   7
   5 6 8 1
```

FACTORIALS!—The number of zeros at the end of a factorial is the same as the power of 5 when the factorial is written as the product of prime factors.

WHEELS WITHIN WHEELS—The small circle is one-ninth the area of the large circle. The distance between the centres is twice the radius of the small circle.

B.A.

TRAFFIC STATISTICS

It has been rightly said that you can prove anything with statistics, provided they are abused. The following letter to a national newspaper is a good example of their abuse.

"It has been suggested that in view of recent motorway accidents a speed limit of 70 m.p.h. on motorways should be imposed. Few drivers realize that time savings are not constant for equal velocity increments. Over a journey of 10 miles, to increase speed from 10 m.p.h. to 20 m.p.h. saves 30 minutes ; from 30 to 40 m.p.h. saves five minutes ; from 50 to 60 m.p.h. saves two minutes. The total time saved by increasing speed from 60 to 100 m.p.h. is only four minutes.

A simple graph of these speed/time values shows quite clearly that any speed over 55 m.p.h. is of value only as an emotional experience for the driver."

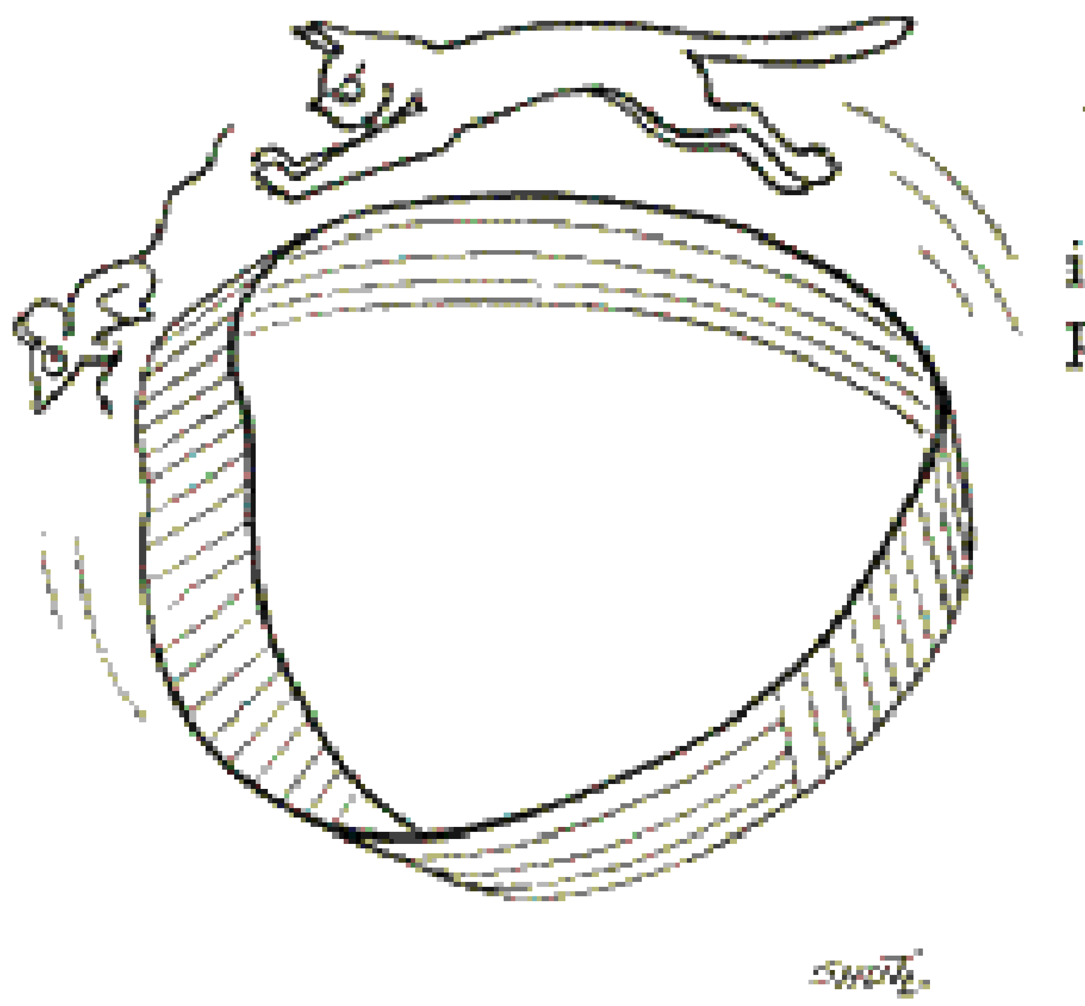
Whilst the figures given are correct, it does not even require a simple graph to spot some of the fallacies in the conclusions drawn by the writer. Firstly, the time taken for *any* journey is halved by doubling the speed, irrespective of the distance travelled. Secondly, few people only travel 10 miles on a motorway. The conclusions look rather different if you consider the Leicestershire business man who travels about 100 miles down the M1 to London. At 50 m.p.h., the journey will take two hours ; at 100 m.p.h. only one. The saving is rather more than four minutes, now!

Perhaps a more impressive argument against high speeds would be the stopping distances for various speeds given on the back of the Highway Code. In perfect conditions — good driver, good weather, good roads — the stopping distance *S*, (in yards), for a given *V* m.p.h. appears to obey the law $S=\frac{1}{3}\left(\frac{V^2}{20}+V\right)$ for speeds up to 60 m.p.h. Assuming that the law still holds at 100 m.p.h. the distance is 200 yards, whereas it is only $58\frac{1}{3}$ yards at 50 m.p.h. Obviously, therefore, the chances of avoiding any hazard are much lower at 100 m.p.h. Or am I miss-using statistics ?

P.S.—If you want to read a humorous account of the subject, try "How to lie with statistics" by Darrell Huff, published by Gollancz.

S.T.P.

TOPOLOGICAL VERSE



Mr. J. Cameron of Hastings was inspired to wax lyrical on another property of the mobius strip.

On a Mobius strip
A cat took a trip
And passed by a mouse walking slow.
When half way ahead
He wished to be fed
So dug down to his prey from below.

PI MNEMONIC

A mnemonic that will probably appeal to a group of young people. It was submitted by Julian Moss of Rayleigh.

"Yes, I have a super motorbike to travel about the roads foolishly."

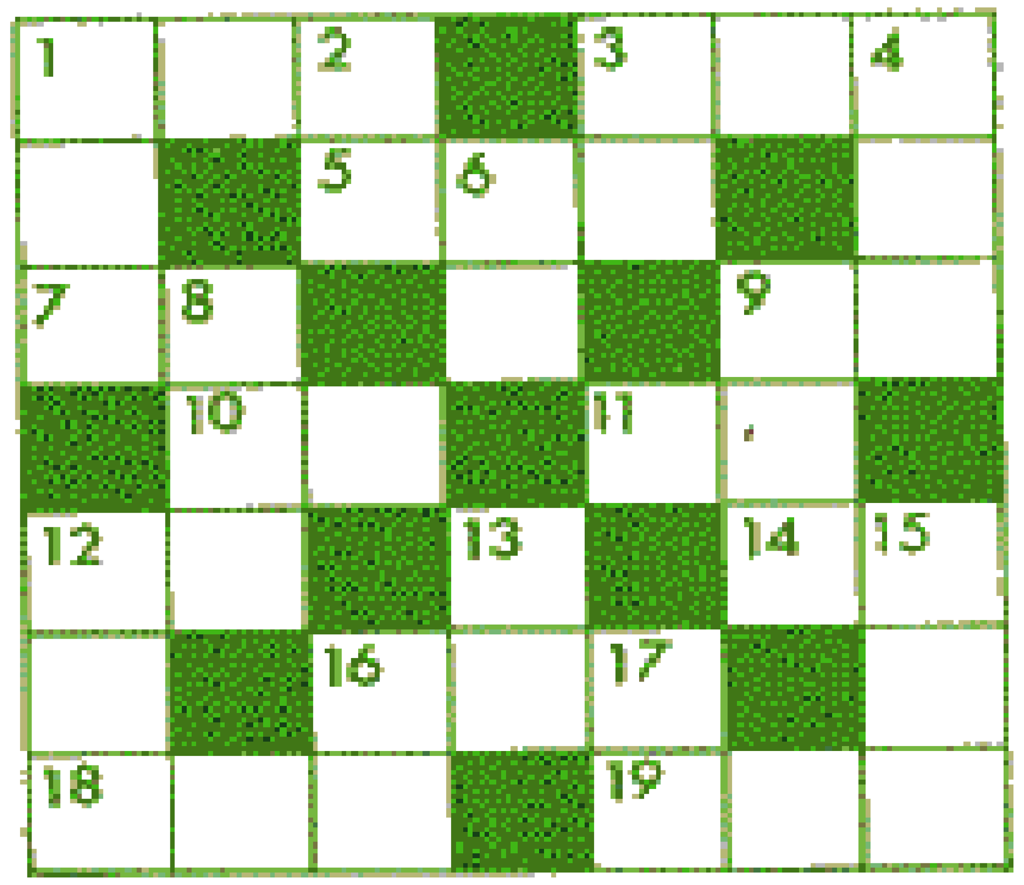
SPOT THE BALL

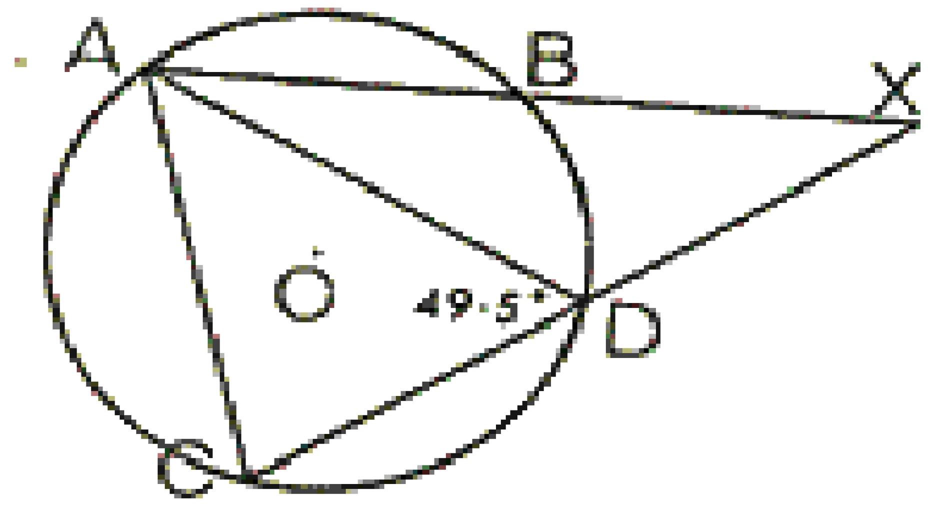
Six spots can be placed on a sphere so that each is the same distance (a quarter of the circumference) from four others.

How many other sets of spots can be spaced symmetrically on a sphere?

C.V.G.

SENIOR CROSS FIGURE No. 55





Clues Across :

1. Intersection, as an ordered pair, of $y=2x^3+3x-24$ and $y=3(x+10)$.
3. The simple interest if a sum invested at 4% per annum amounts to £38.92 after 3 years.
5. Length CX in diagram given only that AB = 92.5 units, BX = 27.5 units and DX = 40 units.
7. Area (sq. mm) of a triangle ABC in which a = 12 mm, b = 32 mm and angle ACB = 30°.
9. Double a prime.
10. across, 13-down and 17-down: The points which satisfy $P \cap Q \cap R$, given that x and y are integers and that
    $P = \{(x,y) : 3y < 17-2x\}$,
    $Q = \{(x,y) : y > x\}$ and
    $R = \{(x,y) : 3y < 7x\}$.
11. Value of x if $\log_5 x = y$ and $\log_2 y = 1$.
12. $\frac{5}{\tan x}$ if $13 \sin x = 5$.
14. Degrees longitude separating two places which are 2610 km apart and on the same 60° latitude. (Radius of earth = 6370 km).
16. Angle (degrees) through which the hour-hand of a clock turns from 15.50 hr. to 20.14 hr.
18. Volume (sq. units) generated when $4x^2+4y^2=49$ is rotated about the x axis.
19. Tan.258°.

Clues Down :

1. Major arc AC in diagram if radius of circle is 70 mm.
2. Value of g in $T=2\pi\sqrt{\frac{l}{g}}$ when T = 4, l = 3.97 and π = 3.142.
3. Distance of chord AC from centre "O." (Ans. in mm).
4. Tax paid at 41p in £1 on interest on £300 at a rate of 6%.
6. Probability of scoring 4 or a multiple of 4 when 2 dice are thrown.
8. (x,y,z) if $x+y+z=9$; $x-y+z=7$ and $x+y-z=5$.
9. Net rate per cent of interest in 4-down when tax has been deducted.
12. $p \propto \frac{1}{v}$ and $p=9\frac{1}{2}$ when $v=4$. Calculate v when $p=\frac{2}{3}$.
13. See 10-across.
15. $\vec{OA} \cdot \vec{OB}$ if $\vec{OA}=(3,4)$, $\vec{OB}=(1,1)$
16. Velocity (m/sec) if distance is given by $s=ut-\frac{1}{2}at^2$ and u = 59 m/sec, t = 5 sec and a = 9.800 m/sec².
17. See 10-across.

D.I.B.

DECIMAL CURRENCY

Most of you know that in 1971 our currency will "go decimal" and that by 1975 we transfer from our traditional hotch-potch of weights and measures to the Metric System. It is estimated that the average child spends about $1\frac{1}{2}$ years of his junior school life learning to cope with our present system with its twelves and twenties, threes, twenty-fours, 1760, 112, 14, 16, and 8. It will be possible to spend that year and a half more profitably from now on. There are much more interesting parts of mathematics to explore.

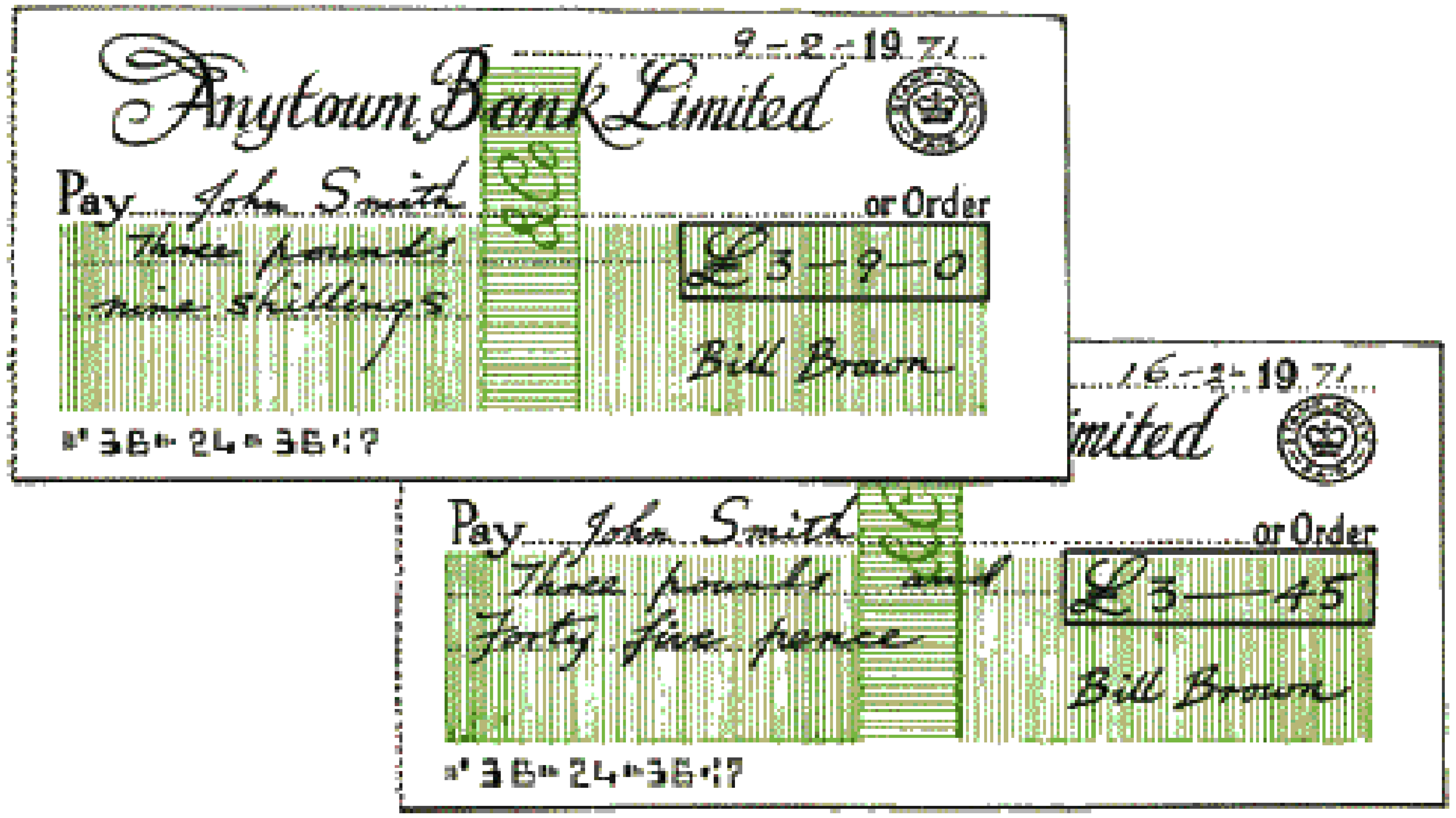
DECIMALISATION PROGRAMME

DATE	NEW COIN ISSUED	OLD COIN WITHDRAWN (i.e. demonetised)
1968	5p, 10p	Shilling and florin are gradually being replaced
Aug. 1st 1969	–	$\frac{1}{2}$d
Oct. 1969	50p	Ten shilling notes withdrawn as they wear out
Jan 1st 1970	–	Half crown
DECIMAL DAY Feb. 15th 1971	$\frac{1}{2}$p, 1p, 2p	–
As soon as possible after D. Day	–	1d, 3d
Later (within 3 – 6 months)	–	6d

The time-table for decimalisation has been published. We have three of the coins in circulation already, the 5 and 10 new pence pieces and the seven-sided 50p coin. This coin has curved sides and is a "curve of constant breadth" (see Pie issues No. 39 and 40). This feature is to enable it to be used in slot machines; perhaps before long we shall be able to buy petrol from a slot machine pump at any hour of the day or night.

The average life of a ten shilling note is only about 5 months and it is estimated that the life of one of the new 50p coins will be 40 to 50 years, so that the reason for its introduction is not hard to find. You may have noticed that far more 10p pieces are taking the place of the florins in your change and that fewer halfcrowns were about even before it was demonetised in January. The banks called in the halfcrowns for re-minting into 10p pieces. The half-penny has disappeared and no more new coins will be issued until Decimal Day, February 15th, 1971, when the new $\frac{1}{2}$p, 1p and 2p coins (which we have already seen in presentation sets) will come into use. The changeover period will probably take some months while slot machines and cash registers are altered but as soon as possible after D Day, the 1d. and 3d. pieces are to be withdrawn and later the 6d. piece. This is the smallest old coin that has an exact new coin equivalent.

You will probably be plagued with conversion tables and calculators aiming to convert from one system to the other. If you are wise, you will ignore them completely and *learn to think entirely in the new system* and what it will buy. The old system will have gone; don't try to live in the past, your place is in the future with the new coinage.



When writing about new coinage, we must be careful to be consistent. We can talk about 97p or £0.97. We *must* write 80p as £0.80 NOT £0.8 and 8p as £0.08 to avoid confusion, using two decimal places for the new pence when expressed in pounds. The Decimal Commission have declared, in their wisdom (sic), that the new $\frac{1}{2}$p is to be written in fraction form, i.e., $37\frac{1}{2}$p is to be £$0.37\frac{1}{2}$ NOT £0.375, which would be more logical. The use of mixed units is not recommended.

R.M.S.

A NINETEENTH CENTURY SQUARE

Late in the last century, a man declared that his age was the square root of the year in which he was born. In what year did he say this?

R.H.C.