GRAPHS

Very often, when drawing graphs, people tend to forget the effects of choosing different scales, and this sometimes leads to arguments between people whose graphs are both correct but appear to be different. To emphasise what a difference the choice of scale might make, let us try a rather unusual way of marking out the x and y axes : starting from the bottom left hand corner O of a piece of paper (cm. ruling graph paper) draw the x and y axes as usual. Now for the scales—mark 1 on the x-axis 8 cm. from O ; mark 2 at a distance of 4 cm. from 1 ; 3 at a distance 2 cm. from 2 ; 4 at a distance 1 cm. from 3 ; . . . . What is happening to the distance each time ? Mark the numbers 5, 6, and 7 at the appropriate places. Where would 8 be ? (Do NOT try to get it in!) What about 9, 10, 11, . . . ? 100 ? 10 000 ?

It can be shown that $8+4+2+1+0.5+0.25+\ldots\ldots$ never exceeds 16, so that all the numbers from 4 upwards are squeezed in between 15 cm. and 16 cm. from O. As there are bigger gaps between O and 1, 1 and 2, and 3 and 4, it would be useful to put in points for $\frac{1}{2}$, $1\frac{1}{2}$, $2\frac{1}{2}$. The distance between O and $\frac{1}{2}$ must be twice the distance bewteen $\frac{1}{2}$ and 1. If we call the latter distance d cm., we have $2d+d=8$, so d is $2\frac{2}{3}$. This is the necessary distance *between* $\frac{1}{2}$ and 1. Mark in the $\frac{1}{2}$ point and use similar reasoning to fix the positions of $1\frac{1}{2}$ and $2\frac{1}{2}$. Mark the y-axis in the same way as the x-axis and we are now ready to plot some graphs.

Consider the pairs of numbers which fit the equation $y=2x$; some examples are ($\frac{1}{2}$, 1), (1,2) ($1\frac{1}{2}$, 3). Plot the points described by these pairs. What were you expecting ? What has happened ?

Plot the graphs of $y=3x$, $y=\frac{1}{2}x$, and $y=\frac{1}{3}x$. Try $y=4-x$, $y=2-x$, and $y=\frac{1}{x}$. Perhaps you would like to think up some equations of your own and try them ?

E.G.

" Smashing bird across the way—3, 5 "

SHORT PUZZLES

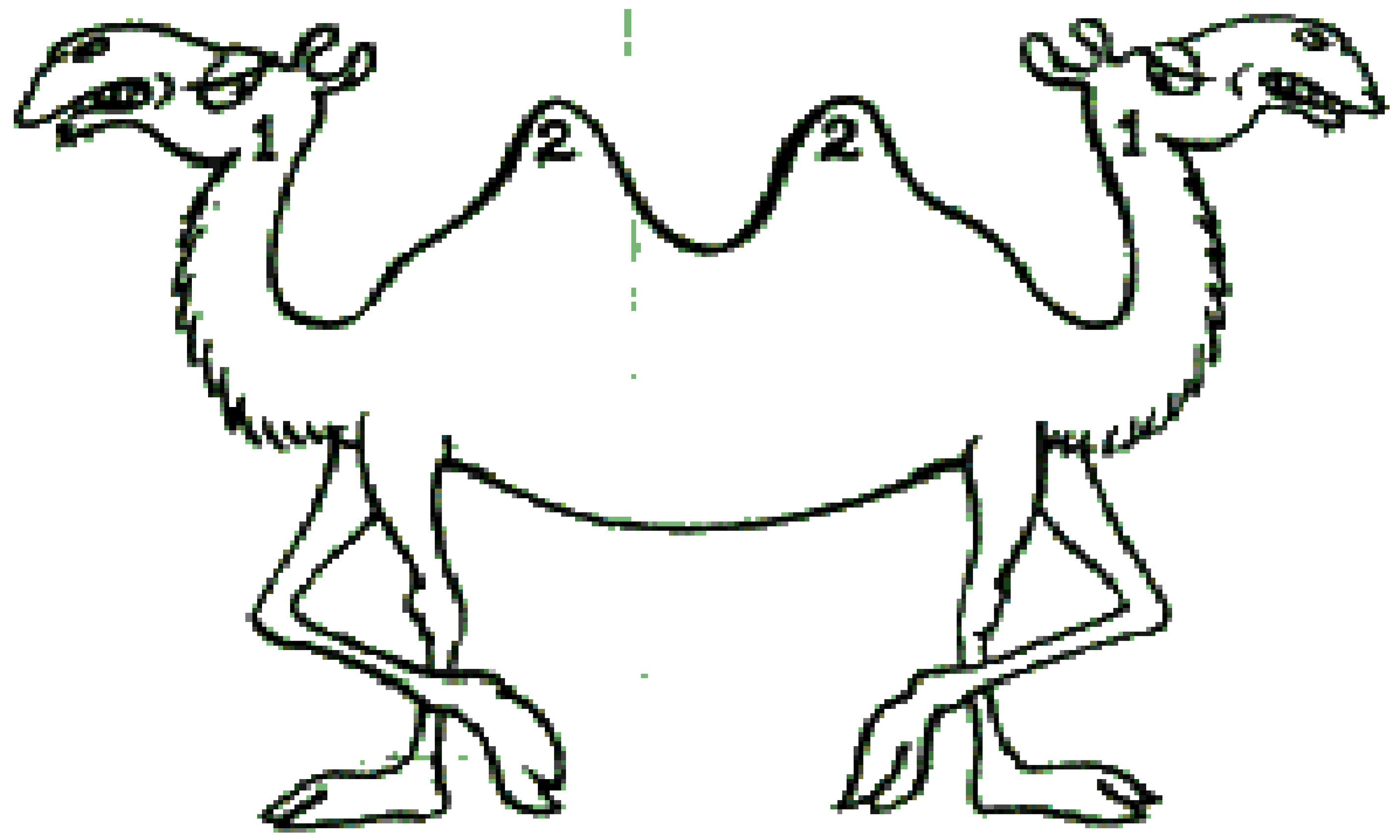
1. Place 4 nines in such a manner that they will make exactly 100.
2. What is the value of $4^{3^{2n}}$ when $n=0$. ?

mathematical pie

No. 61 *Editorial Address :* 100, Burman Road, Shirley, Solihull, Warwickshire, England AUTUMN, 1970

PALINDROMES

You probably know that a palindrome is a word which reads the same backwards as forwards, such as the name ANNA or the phrase supposed to have been used by Adam to Eve : "MADAM I'M ADAM ", or Napoleon's phrase " ABLE WAS I ERE I SAW ELBA ". (Unfortunately I somehow do not think that either Adam or Napoleon spoke in English!) When Shutl. heard this, his imagination got the better of him and he produced



The Palindrome-dary or Camelemac

We can consider palindromic numbers such as 14641 which read the same both ways. How many such whole numbers are there between 7 and 1 000 000 ?

R.M.S.

RICHNESS OF LANGUAGE

Shepherds in different parts of the country have had their own system of numeration, although variations occur even in the same county. Wiltshire counting could be yan, tan, tethera, pethera, pimp, etc., or onetherum, twotherum, cocktherum, qutherum, setherum, etc.

The Lincolnshire shepherds and those of the Lake District each have their own sequence of words for the counting numbers which have survived over the years.

R.H.C.

CLOCK ARITHMETIC No. 5
The Remainder of the Story

In issues No. 34, 35, 37 and 60, we talked about combining numbers in clock arithmetic. In the game, we had a clock face numbered from 1 to 6 and illustrated calculations such as $3+2$(clock 6)$=5$, $4+3$ (clock 6)$=1$, $5+5$ (clock 6)$=4$. Now let us add together numbers such as 9 and 16. If we had to deal with the addition of these two numbers on the clock 6, 9 would be represented by 3 whilst 16 would be 4. We arrive at the figure 3 by finding how many times 6 divides into 9 (once) and take the remainder which is 3. Similarly 16 divides by 6 twice and leaves a remainder of 4. This suggests that the remainders of any number when divided by 6 are the representation of that number in clock 6. We call these remainders the modulus of that number based on the particular clock face. For example, $16(\text{mod.}6)=4$, $9(\text{mod.}6)=3$.

If we now return to the addition of 9 and 16 on the clock 6 face, the answer is 1. Is it a fluke that the answer arrived at could be written this way?

$$16 \quad +9 \quad =25$$
$$16\ (\text{mod.}6)+9\ (\text{mod.}6)=4+3=7=1$$
$$=25\ (\text{mod.}6)$$

But is it a fluke? is $17\ (\text{mod.}6)+8\ (\text{mod.}6)=25\ (\text{mod.}6)$?

More generally, is $x\ (\text{mod.}6)+y\ (\text{mod.}6)=(x+y)\ (\text{mod.}6)$?

is $x\ (\text{mod.}6)-y\ (\text{mod.}6)=(x-y)\ (\text{mod.}6)$?

Now investigate the operations of multiplication and division.

$x\ (\text{mod.}6)$ times $y\ (\text{mod.}6)=xy\ (\text{mod.}6)$, $x\ (\text{mod.}6)\div y\ (\text{mod.}6)=\frac{x}{y}\ (\text{mod.}6)$.

What about $[x\ (\text{mod.}6)]^n=x^n\ (\text{mod.}6)$?

You should now have a theory about modulus 6 arithmetic. Test it in a different clock face base. R.H.C.

CHARLIE COOK AGAIN

On his Summer Examination Paper, Charlie Cook wrote down

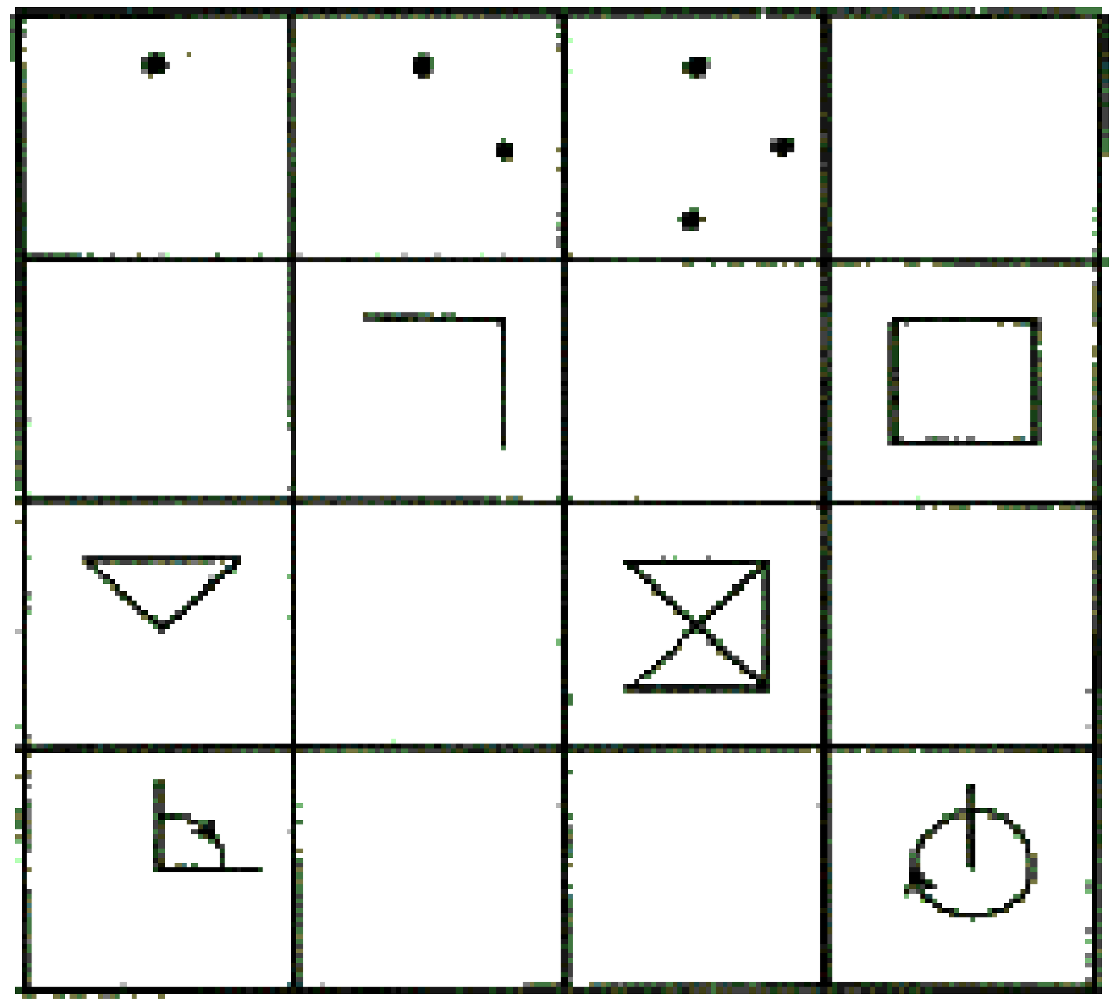
$$\frac{3}{7} + \frac{4}{5} = \frac{7}{12}$$

and he was correct! Can you explain this statement? D.I.B.

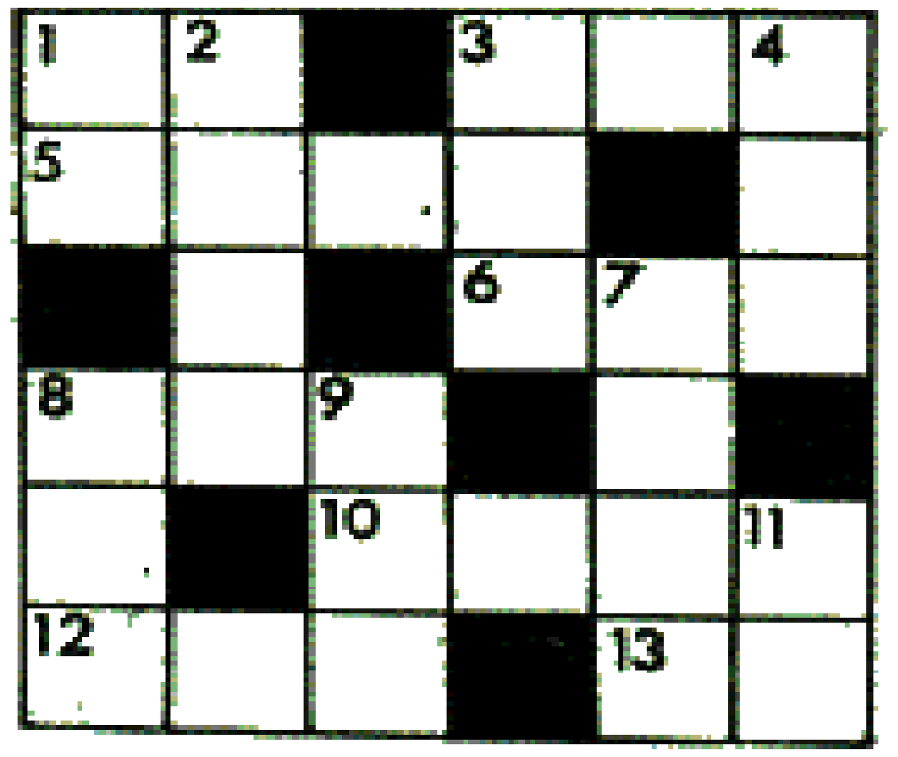
SEQUENCES

The block shows four sequences which are connected both horizontally and vertically. Can you complete the diagram.

E.G.



JUNIOR CROSSFIGURE No. 53



Clues Down

1. First odd square with two digits.
2. $2^{12}+2^7$.
3. Circumference (in inches) of a circle of diameter 5′ 3″, take π to be 22/7.
4. James Bond?
7. 1 ton, 1 cwt., 1 qr., 1 lb. in lb.
8. 5^3.
9. Centimetres in $4\frac{1}{4}$ metres.
11. 5 times (13 across).

Clues Across

1. Product of the first four positive integers.
3. New pence in one Pound.
5. One less than the number of feet in 1 mile.
6. $(10^2-1)\times 3^2-4^3$.
8. One gross.
10. Product of the 80's primes.
12. 55^2-50^2.
13. One fifth of (11 down).

P.J.G.

SOLUTIONS TO PROBLEMS IN ISSUE No. 60

Although great care is taken with the issue, misprints are sometimes missed. The editor will send a book token to the first reader who recognises a misprint and writes in to suggest what the original should have been or how the misprint can be corrected.

SENIOR CROSS FIGURE No. 56

Clues Across: 2. 592; 5. 245; 7. 784; 9. 71; 11. 95; 12. 36; 13. 24; 14. 90; 15. 32; 16. 961; 18. 385; 19. 137.

Clues Down: 1. 94; 3. 97; 4. 289; 5. 24389; 6. 57; 8. 45825; 10. 120; 14. 91; 15. 387; 18. 33. The clue to 17 down lead to an impossible situation as the slant height should have read 18 ft. 9 in.

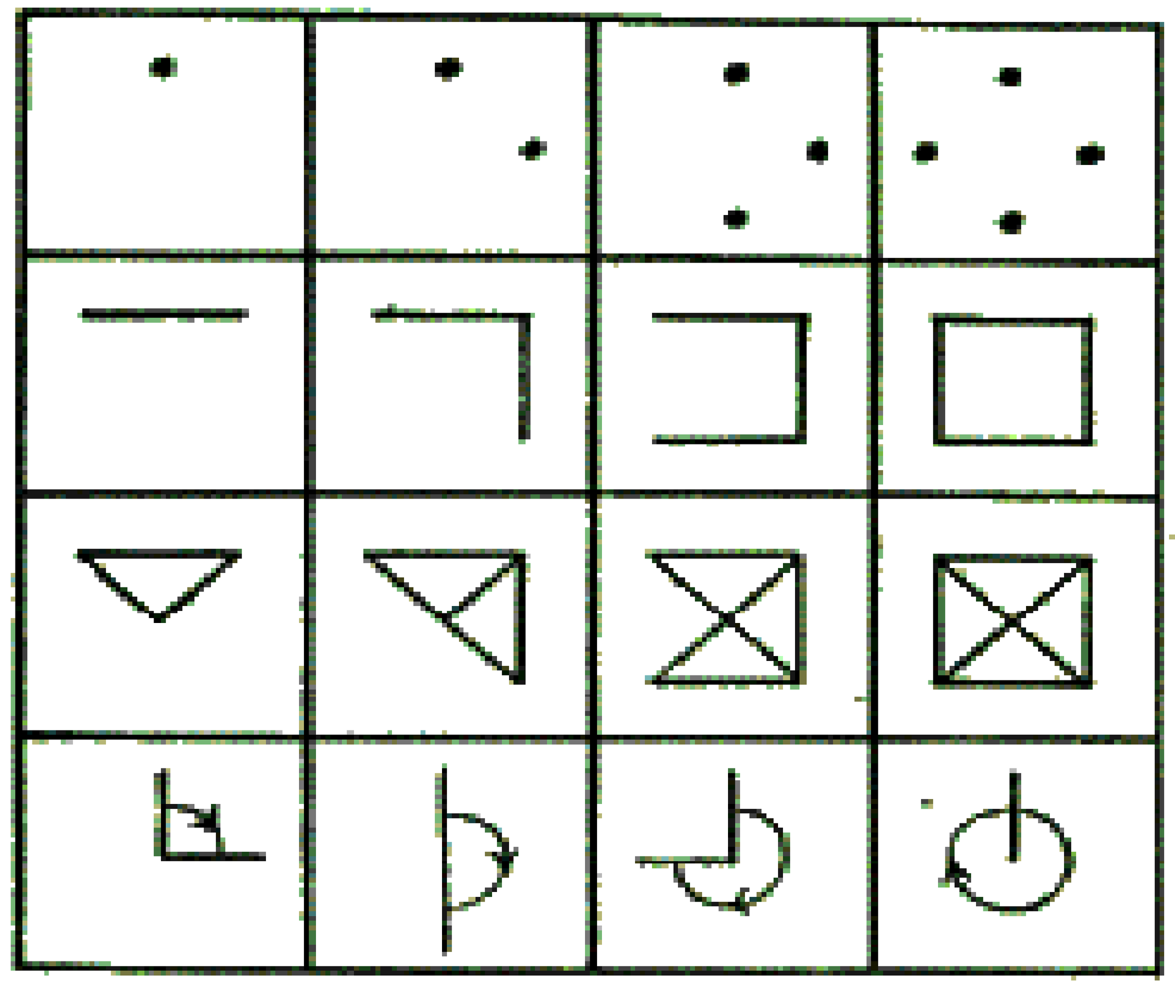
JUNIOR CROSS FIGURE No. 52

Clues Across: 2. 15; 5. 3142; 7. 17; 9. 227; 10. 111; 12. 46; 13. 34; 15. 29791.

Clues Down: 1. 131072, this is incompatible with 10 across; 2. 13; 3. 512; 4. 32768; 6. 424; 8. 71; 11. 137; 14. 49.

PAIRING UP—A number of suggestions have been received for solving the problem, the best solution will be given in issue No. 62.

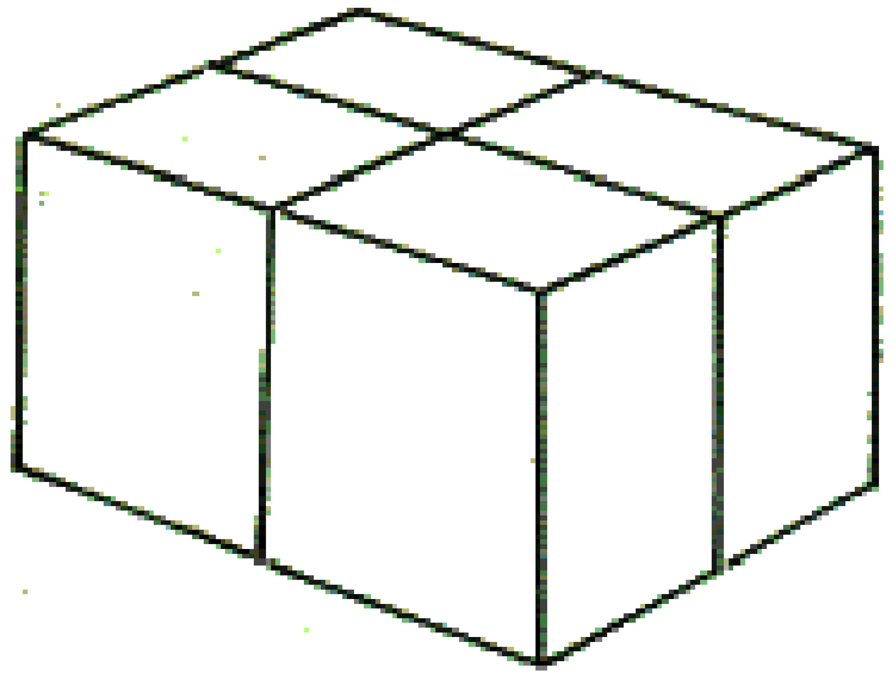
SEQUENCES—In issue No. 61 page 478.

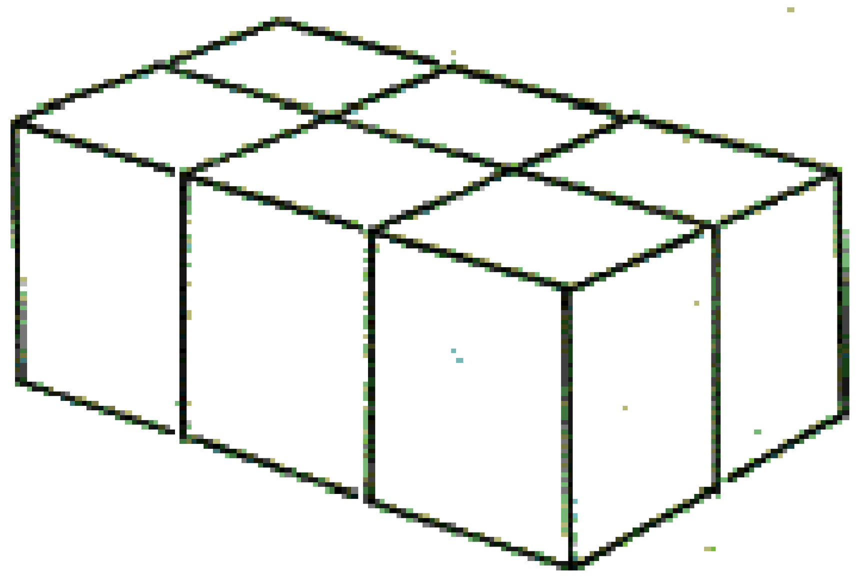


B.A.

WRAP UP!

When tying up a Christmas parcel, I have twelve feet of string (exclusive of knots). If I have to pass it round as in the left hand diagram find what dimensions should be used in order to give the greatest volume.





For another parcel, I have to pass the same length of string once lengthwise and twice round its girth, as shown in the right hand diagram. What are the dimensions that give the greatest volume now?

R.H.C.

ALL THE SIXES

Stifel (1485-1567) was one of the oddest personalities in the history of Mathematics. He was originally a monk, was converted by Martin Luther and became a fanatical reformer. His erratic mind led him to indulge in number mysticism amongst which he concluded that Pope Leo X was the 'beast' mentioned in the book of Revelation.

"Let him that hath understanding count the number of the beast; for it is the number of a man; and his number is six hundred, three score and six".

Stifel's argument ran thus:

LEO X LEO DECIMUS LDCIMV (omitting E, O, S), add X and omit M because M stood for 'mystery' LDCIVX. Can you now reorganise this to give 666? Q.E.D.

GOING DECIMAL!

" Even the parrot's gone decimal, he keeps shouting "Pieces of ten."

" Ever since the firm went metric you've come in point five sloshed after lunch!"

A BASE QUESTION No. 2

In issue number 60, we considered three digit numbers with the property that the order of the digits was reversed when the number was changed from one base to another.

$abc_{ten} = cba_{nine}$ when $a = b = 4$ and $c = 5$, $abc_{eight} = cba_{seven}$ when $a = b = 3$ and $c = 4$, $abc_{six} = cba_{five}$ when $a = b = 2$ and $c = 3$ and $abc_{four} = cba_{three}$ when $a = b = 1$ and $c = 2$.

$abc_{twelve} = cba_{eleven}$ when $a = b = 5$ and $c = 6$. How far can the relation be extended upwards before new symbols must be introduced? Did you notice that the higher base was even and the lower base odd? Is it possible for the converse to apply?

B.A.

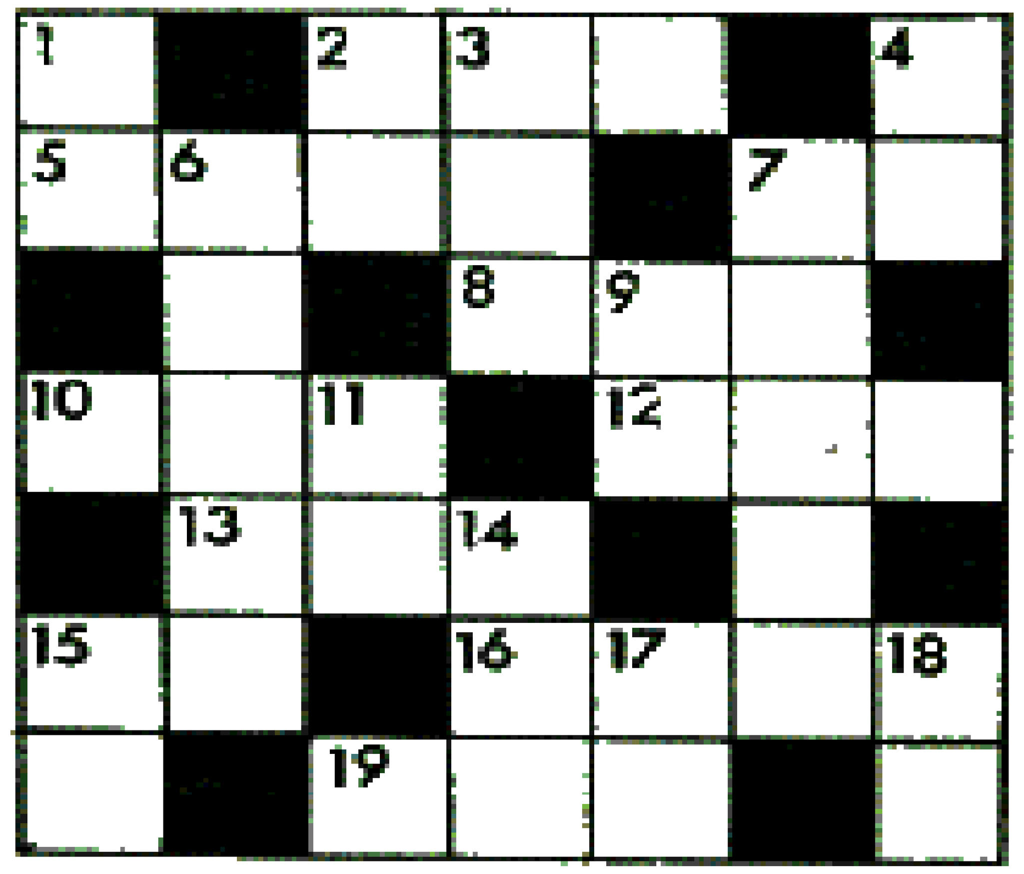
HIGH FINANCE

The largest sum of money involving pounds, shillings, and pence which can be written down using the digits, 1, 2, 3, 4, 5, 6, 7, 8, 9, once only is £9876543 . . 2s. . . 1d. What is the smallest?

SENIOR CROSS FIGURE No. 57

Clues Across

2. 8th term of G.P., 1st term is 6, common ratio is 2.
5. Distance, in n.m., over N. Pole along line of latitude from 50°N to 53°44′N.
7. tan 211°48′.
8. $\sqrt[3]{(450)}$ correct to three sig. fig.
10. (Perpendicular height of a regular tetrahedron of edge 10 cm.)^2.
12. Acceleration due to gravity in cm. per sec.^{2}.
13. Volume of revolution formed by rotating $x^2+y^2=36$ about the x-axis (Take π to be 3.14) x positive.
15. Maximum value of $x^3-6x^2-36x+8$.
16. $\log x^2$ when $\log x$ is 0.1823.
19. Third side of triangle with sides 5 cm., 10 cm., and angle between 120°.



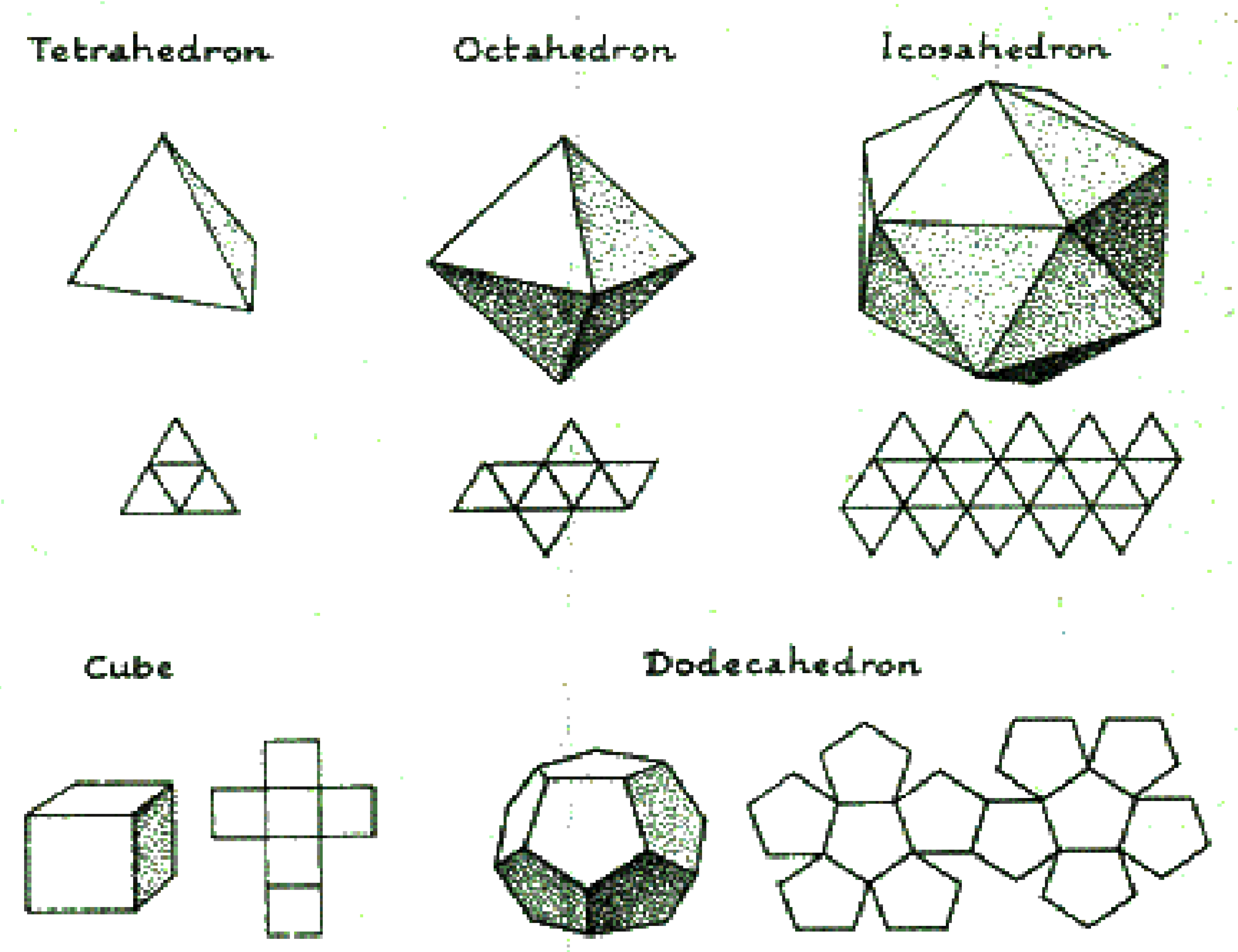
Ignore decimal points.

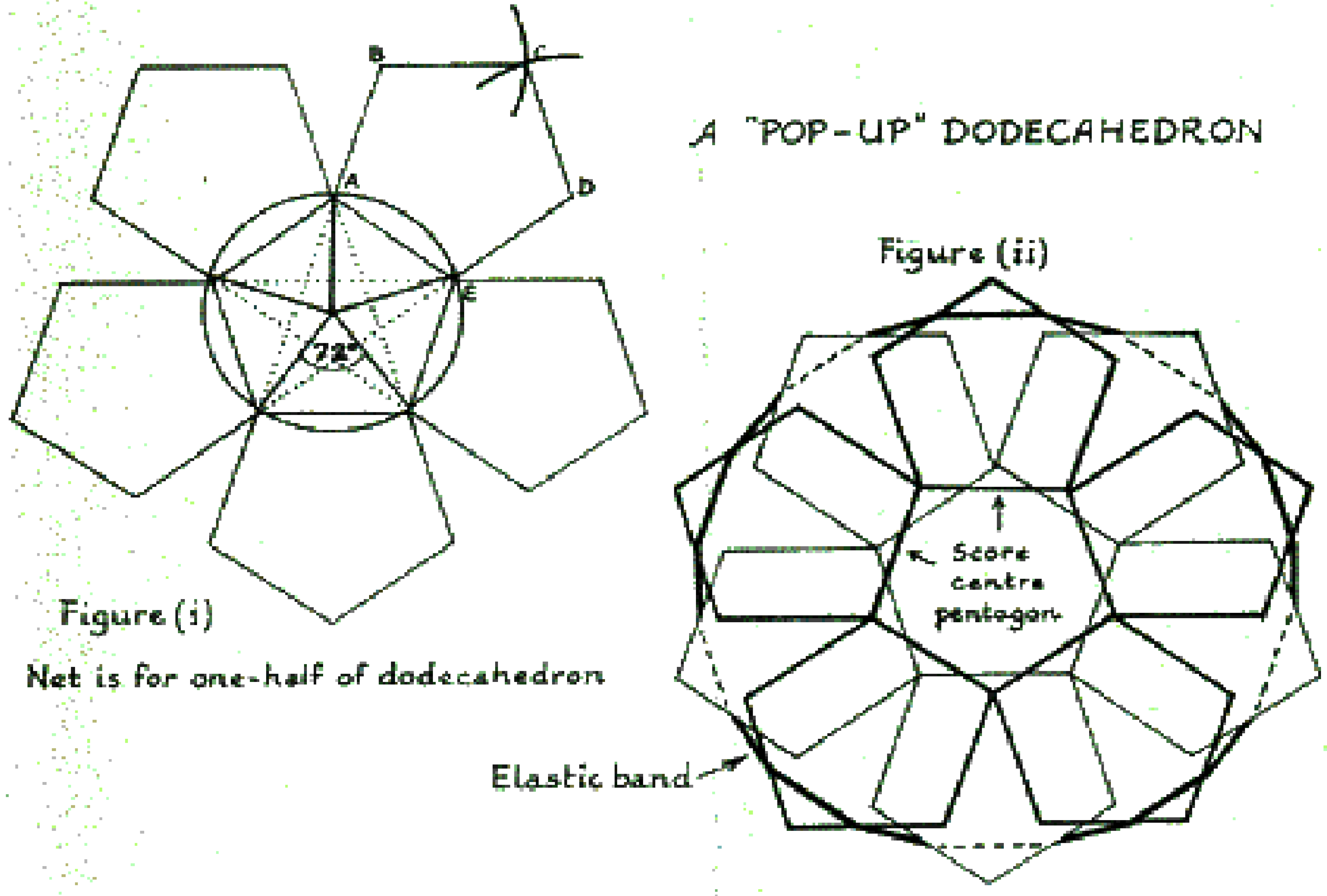
Clues Down

1. Twice the number of faces of a dodecahedron.
2. y is proportional to $\frac{1}{\sqrt{x}}$, $y=2$ when $x=49$; y if x is $\frac{4}{121}$.
3. Largest root of $3x^3+x^2-2x=0$ correct to 3 dec. places.
4. Reverse the digits of the gradient of $y=x(x\sqrt{x}+3)$ at $x=4$.
6. Next two terms in the sequence 100, —3, 85, 18, 70, —108,.
7. Roots of $2x^2-18x+11=0$ correct to 2 dec. places, smaller first.
9. Product of odd primes.
11. Area between $y=3x^2-4x$, the x-axis, $x=0$, and $x=5$.
14. Thirteenth Fibonacci number.
15. $(2.25)^{1.5} \times \left(\frac{27}{64}\right)^{-\frac{2}{3}} \div 2^{-3}$
17. Radius of circumcircle of triangle ABC in which $a=62$ cm., and $\angle A=30°$.
18. Coefficient of x^4 in the expansion of $(1+x\sqrt{2})^6$.

P.J.G.

PYTHAGOREAN SOLIDS





The Pythagoreans (6th Century B.C.) were able to construct the tetrahedron, cube and dodecahedron. The octahedron and icosahedron are thought to have been discovered later by Theætetus (4th Century B.C.) who was a pupil of Plato's Academy. Thus, the five polyhedra (many faces) have become known as the Platonic solids. Each solid is convex and has regular congruent faces with identical vertices. It is not possible to have any other regular solids of this type.

The solids can be constructed simply from card. The nets are shown with some of the construction lines left in to guide you, with the *alphabetical* order of drawing. If you are unable to *construct* a pentagon, start the dodecahedron net by dividing a circle into sectors of 72° with a protractor.

Carefully score all fold lines with the back of a knife or pair of scissors. Stick flaps (ommited from diagrams) on the inside of each model with a quick-drying cement (e.g. Bostik). Two coats of enamel paint will give the solids a most attractive finish.

D.I.B.