AS FAR AS THE EYE CAN SEE

Taking the earth's radius to be 6,370km, what is the distance of the horizon from an observer at an altitude of 100m above sea-level?
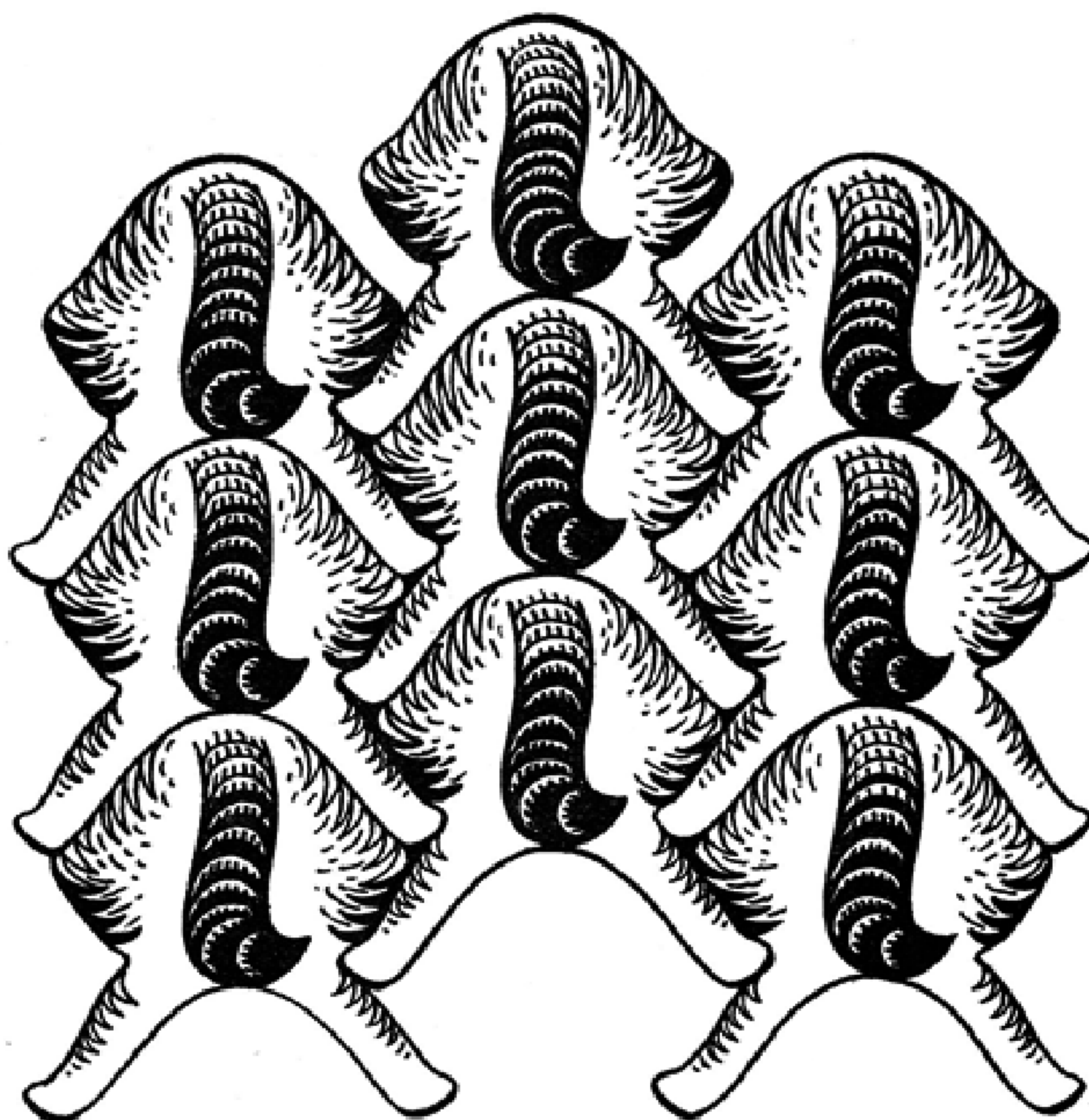
Can you generalise this result and find a formula for this distance when the altitude is known?   D.I.B.

WHAT AM I?

1. I am usually a small number.
2. My appearance can be changed by division without changing my value.
3. Sometimes I am less than 1.
4. Sometimes I am called a ratio.
5. Sometimes I am a percentage.
6. Sometimes I am a decimal.   R.H.C.

FELIX TESSELLATUS

FELIX TESSELLATUS
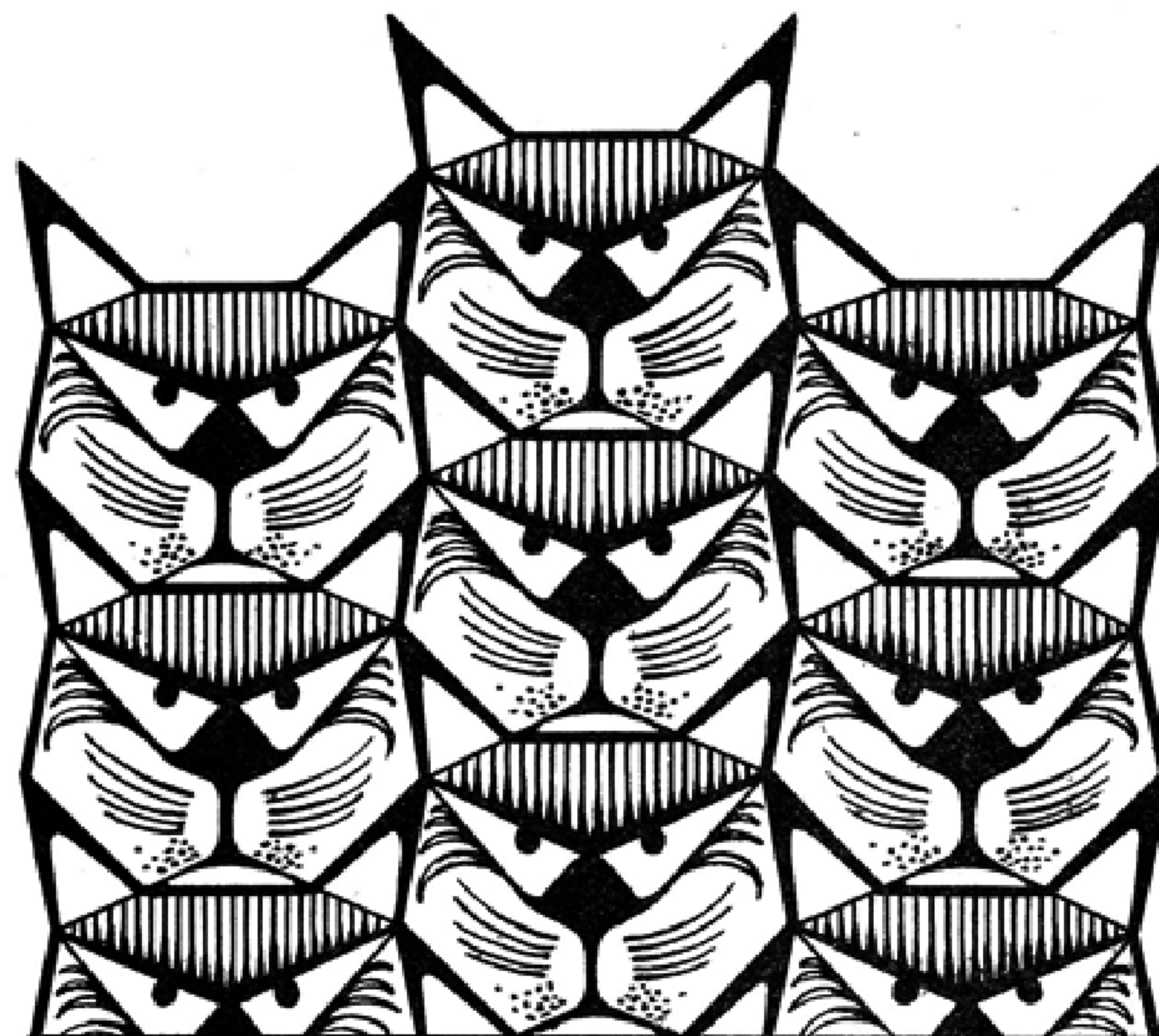
This tessellation was submitted by Christine Wolter of Harlton, Cambs.

HOW LONG IS A PIECE OF STRING?

1.

2.

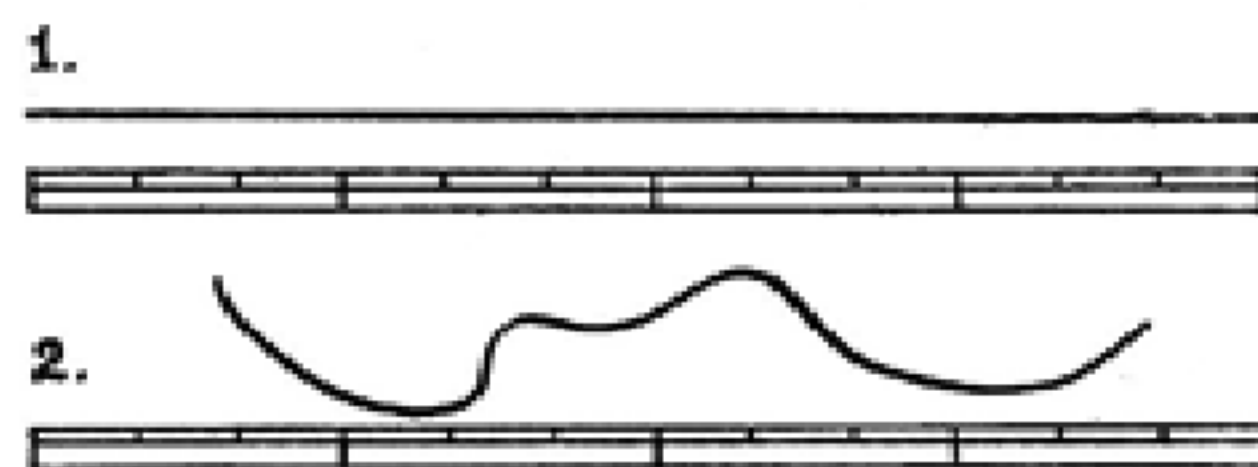A piece of string 12cm long is stretched along the edge of a ruler so that each point of the string can be identified by its corresponding mark on the rule. (In mathematical language we say that the points of the string are in 1 to 1 correspondence with the points of the ruler.)

If we now let go of the string it will, after the nature of pieces of string, tend to double back on itself as shown in the second diagram. If the ends of the string are still somewhere between 0 and 12 on the ruler, it is a fact that there is always at least one point of the string that has the same position as before, i.e., it is opposite the same mark on the ruler. Can you prove it?

(Hint ; plot the new and old positions of the string as graphs against the ruler as the x-axis.)

R.M.S.

ROUND ABOUT 'RITHMETIC

No two digit number in base ten has its digits reversed when doubled. What numbers in bases less than ten have this property?
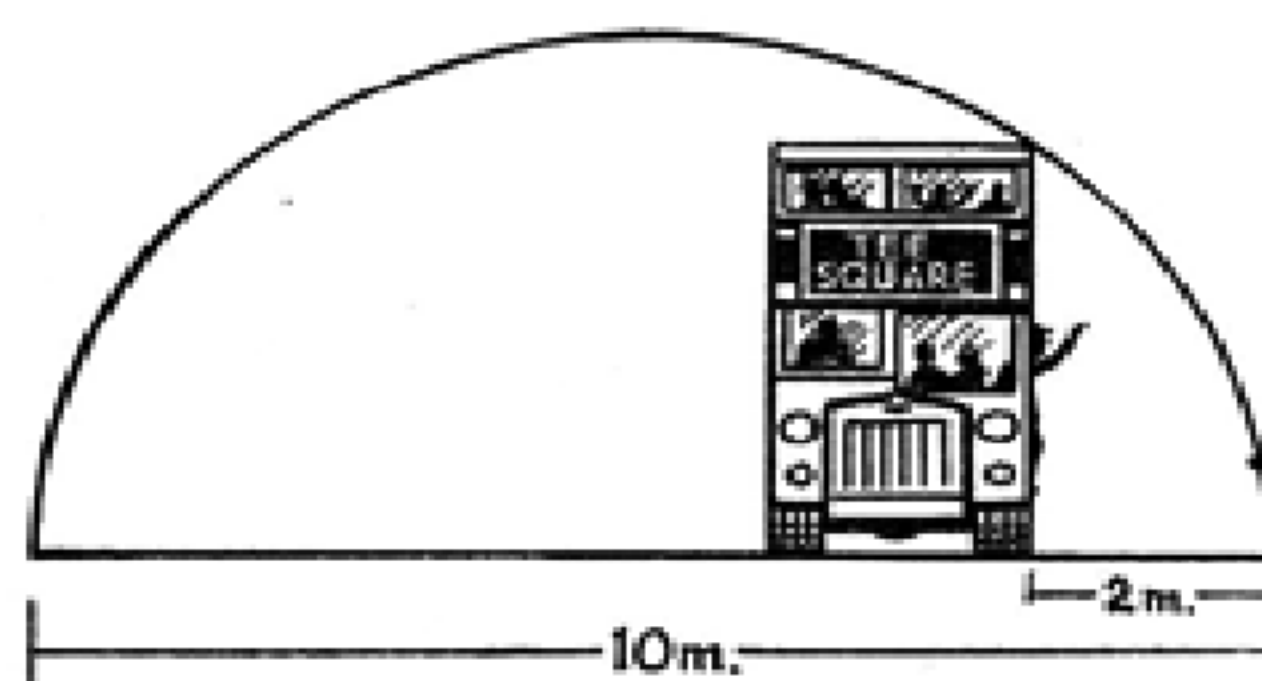
Find a general expression for all bases in which this type of number-pair exists.

Consider the cases when the original number is trebled.

Can you find a rule which gives the least base (b) in which a number can be reversed by multiplication by a given number (n)?

These problems are from an article submitted by Mr. J. Cameron of Hastings.

MIND YOUR HEADS

A tunnel has a semi-circular cross-section of diameter 10 metres. What is the height of a 'bus if its roof just touches the tunnel when the wheels are 2 metres from one side?

D.I.B.

SQUARE THE QUAD

What is the length (in units) of a side of a square which is equal in area to the quadrilateral with vertices $(1, 0)$, $(17, 0)$, $(13, 12)$ and $(0, 7)$?

D.I.B.

WHOOPS!

A rigid circular hoop fits exactly around the 40,000km of the earth's circumference at sea-level. If an extra 10 metres is added to the hoop, at what height above sea-level will it encompass the earth?

A hoop round a barrel is increased in length by 10 metres, what is the increase of the radius of the circle formed by the new hoop?

D.I.B.

JUNIOR CROSS FIGURE No. 59

Take π to be 3·14.

CLUES ACROSS

1. cm in 1 inch.
3. An unlucky multiple of the square of a prime.
5. Next two terms in the series 1, 4, 9, 16, . . .
7. $3^3 + 3^2 + 3$.
9. Remaining angle of an isosceles triangle if the equal angles are 42°.
13. 2/7 as a decimal to four decimal places.
15. Interior angle of a regular 30-sided polygon.
16. xy if $x+y=13$ and $x-y=9$.

CLUES DOWN

1. First prime after ten times first prime after twenty.
2. Number of cards left in a full pack (with jokers) after the picture cards have been removed.
3. CCXII ÷ IV.
4. Anagram of 10 down.
6. Square plus one.
8. $(x-1)(x+1)$ if x is 10.
10. 2π.
11. Number of days in the first five months of a leap year.
12. $(\sqrt{3})^8$.
14. Half a dozen dozen.

P.J.G.

THE DISAPPEARING WINE

3 3 3
3   3
3 3 3

A man had 24 bottles of wine which he arranged as shown. He remembered that the pattern was symmetrical and that there were nine bottles along each side of the square cellar. His wine steward drank from some of the bottles and rearranged the rest in such a way that the master was unaware of the change as the original conditions were fulfilled. What was the new pattern and how many bottles did the wine steward take?

R.H.C.

SOLUTIONS TO PROBLEMS IN ISSUE No. 66

CAREFUL PACKING
The diagonal of a foot cube is 1·73 feet.

NO SEVEN UP
(i) 48, (ii) 78.

CHARLIE COOK
The answer was wrong, but the wrong working would give the correct answer for $\frac{\log 2}{\log 4}$.

GREATEST PRODUCT
64, when X and Y are equal.

NUMBER SYSTEMS
644 in base five, 8 877 690 in base ten.

A FAMILY RELATIONSHIP
4 boys and 3 girls.

SENIOR CROSS FIGURE No. 62
Clues Across : 1. 13120 ; 6. 202 ; 8. 31 ; 10. 25 ; 11. 322 ; 12. 247 ; 13. 65 ; 15. 10 ; 16. 174 ; 18. 98697.
Clues Down : 2. 32 ; 3. 100 ; 4. 22 ; 5. 53361 ; 7. 15708 ; 9. 125 ; 10. 241 ; 14. 176 ; 16. 18 ; 17. 49.

MULTIPLICATION PROBLEMS
$5\sqrt{(60)}+5$ and $5\sqrt{(60)}-5$.
17 in base nine.

JUNIOR CROSS FIGURE No. 58
Clues Across : 1. 27 ; 3. 366 ; 5. 2240 ; 6. 394 ; 8. 251 ; 10. 0791 ; 12. 750 ; 13. 30.

Clues Down : 1. 22 ; 2. 7225 ; 3. 303 ; 4. 664 ; 7. 9993 ; 8. 247 ; 9. 100 ; 11. 10.

B.A.

FOR CRACK MATHEMATICIANS ONLY — 2

The coded message in our last issue read, 'A plane surface is that in which any two points being taken, the straight line between them lies wholly in the surface', which was Euclid's definition of a plane. The large-scale investigation referred to produced the following results for every 2,000 letters:

E	T	A	O	I	S	N	R	H	L	D	C	U	M	F	W	G
246	182	164	146	136	136	134	116	112	87	71	57	55	55	47	47	41

P	Y	B	V	K	X	J	Q	Z
41	39	36	22	16	4	4	4	2

You will see that it divides into three clearly defined groups, and although you may have a slightly different order in each group, it is unlikely that any letter will have strayed into the wrong group. If you used it to decode the message you will probably have been surprised to find how closely the order agreed, even with such a few letters.

Many word-building games are also based on this distribution. If you played 'Scrabble', which has 98 letters and two blanks (standing for any letter) you might like to divide the above results by 20, and compare the figures obtained with the number of each letter in the game. You will find they agree closely for most letters. If there appears to be a discrepancy, particularly in the first group, don't shoot the manufacturer! Remember that there would need to be about 500 letters to jsutify having even one X, J, Q or Z.

S.T.P.

LETTERS TO THE EDITOR

The Editor wishes to acknowledge a number of letters he has received on a variety of subjects.

" Number systems " issue No. 66 should have read " contain each digit " Miss M. Farrant, Southsea, E. Broadhurst, Harrogate.

Mr. J. Perrin, Chatham, has pointed out the error in the solution given in the issue No. 66 to the Modern Cross Figure issue No. 65, "6 across $3\frac{1}{2}$ in binary should be 111 not 1112, whilst Miss Jones, Salisbury, pointed out the spelling mistake " Adross " in the same line as the $3\frac{1}{2}$ error.

The following called attention to the error 1475 (should be 1575) in Multiplication Problem, issue No. 66 : Misses C. Meekan & J. Darby, Smethwick, M. Lee, Lingfield, Miss C. Riley & P. Butler, Melton Mowbray, H. L. Kotkin, London, J. Thornton, Sutton Coldfield, M. Webster, Scunthorpe, Miss M. Farrant, Southsea, S. Hagan, Co. Antrim, Mr. C. Pyle, Southampton.

Miss Morag McCreadie, Hale, pointed out that we had used the same problem " Cutting the mint " in issue 66 and issue 65.

A. Mountifield, Gosport, sent us his " Thoughts on Noughts ", also M.Lt. E. Simon Fitzpatrick, Maidstone, wrote about clue 8 across in Junior Cross Figure, issue No. 66, but he had made a mistake in his reasoning. Hugh Williams and others told us of our mistake in the answer, issue 65, to Junior Cross Figure No. 56, issue 64, 10 down " Palindromic Jumbo Jet " which should be 747 not 707. On the other hand, Miss Annette Brown and Carole Leslie, Dundee and Mr. C. Appleton, Oldham, were taken in by P.J.G.'s clue 9 down Junior Cross Figure, issue 65.
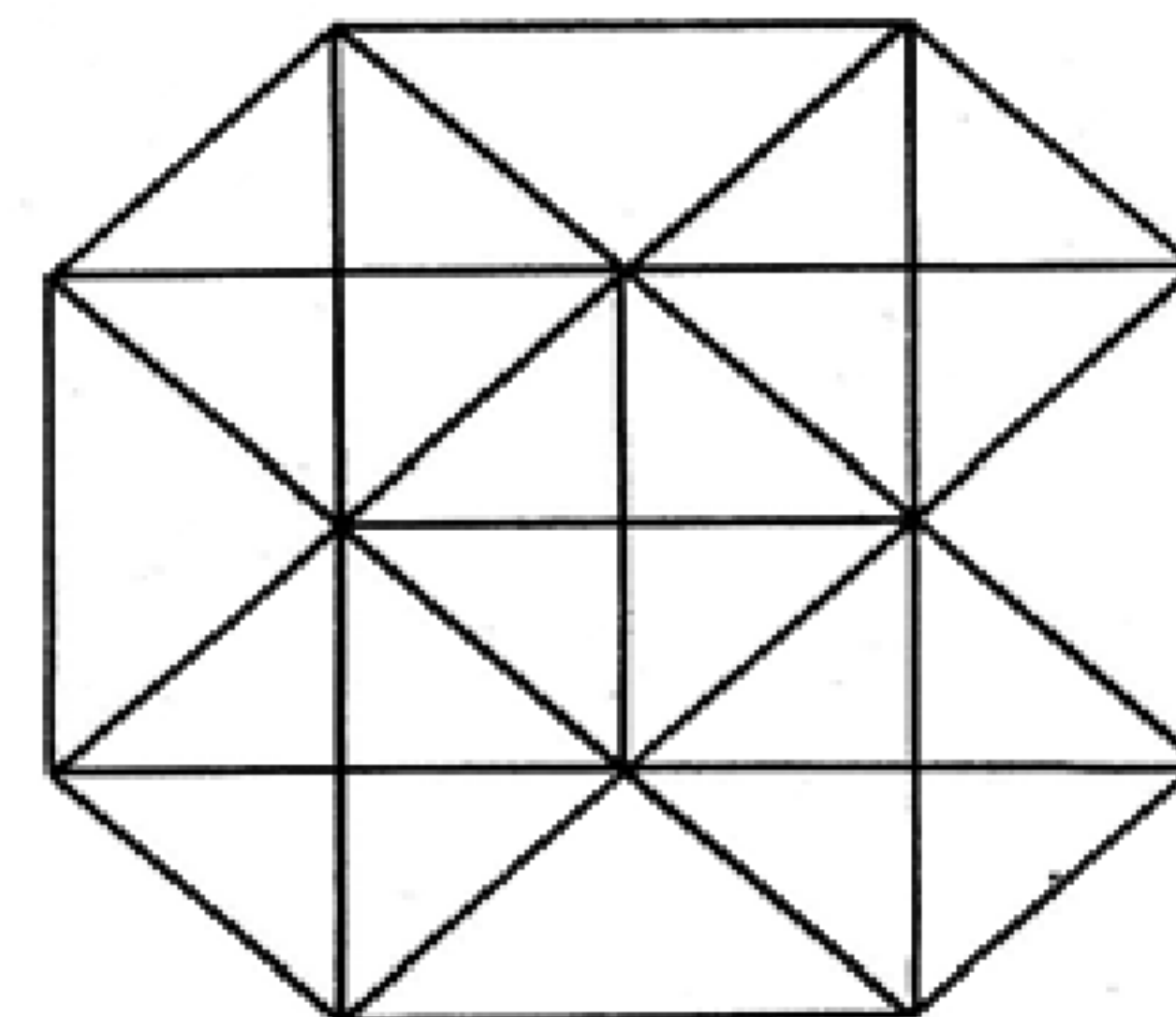
P. Rawles, Tonbridge, queried the answer given in issue 66 to 1 down in the Senior Cross Figure No. 61, whilst G. Middleton, Lincoln, questioned the answers to 10 across and 2 down in the same cross figure.

SORT IT OUT

Rearrange the symbols to make mathematical statement.

CUBICS

How many cubes are shown in the diagram? B.A.

SENIOR CROSS FIGURE No. 63

Ignore decimal points.

CLUES ACROSS

1. y is inversely proportional to the cube root of x. y is 39 when x is $1 \cdot 25 \times 10^2$, find y if x is 8×10^{-3}.

2. $p+q+r$ if $(a^3 b^{-2} c^{-\frac{1}{2}}) \div (a^{-\frac{1}{2}} b^{-2\frac{1}{2}})$ $= \dfrac{a^p b^q}{c^r}$.

5. Maximum value of $2x^3 - 3x^2 - 36x + 12$.

6. $\cfrac{1}{2 + \cfrac{1}{3 + \cfrac{1}{4 + \frac{1}{6}}}}$

CLUES DOWN

1. Larger root of $x^2 - 4x - 47 = 0$.

3. Sixth term of the series (1×2), (4×3), (9×4), ...

4. Coefficient of x^5 in $(1+x)^8$.

7. First palindromic prime greater than 300.

8. Reciprocal of 2·02.

10. $\log_p(\frac{1}{4} p^2)$ if $\log_p 2$ is 0·6786.

11. Smaller of two consecutive odd numbers whose squares differ by 560.

13. Cube root of $x^3 - 3x^2 + 3x - 1$ if x is 30. P.J.G.

8. Area between the graphs $y = x^2 - 2x + 6$, $y = 2x - 3x^2 - 2$, x=0, and x=3.

9. Sum of the longer sides of a right-angled triangle whose shortest side is 13, and the longer sides differ by one.

11. Gradient at (1,5) of $y = 4x^3 + x^2 - x + 1$.

12. $\dfrac{1}{\sqrt{(\pi)}}$.

13. a, if $(3x-2)$ is a factor of $9x^4 - 6x^3 - 3x^2 + ax - 14$.

14. Difference of two cubes, the difference of the original numbers being one-third of the digit sum of the answer.

15. 43 rd term of A.P. whose third term is 8 and whose seventh term is four times the second term.

DOCUMENT

MAGNETIC DISC

To Solve $ax^2 + bx + c = 0$

START

INPUT & STORE
a, b, c

CALCULATE
$b^2 - 4ac$

CALCULATE
SQUARE ROOT OF
$b^2 - 4ac$

CALCULATE
$\dfrac{-b \pm b^2 - 4ac}{2a}$

OUTPUT THESE
TWO NUMBERS
AS THE ROOTS

STOP

In the last issue, No. 66, we considered flowcharts which just used the 4 basic symbols for arithmetical operations; a SYSTEMS FLOWCHART can extend the process to more complicated problems. We shall consider the initial flowchart of a typical operation, the calculation of a gas bill. Several new symbols are used and these are shown on the left whilst the actual flowchart is shown on the right of the page.
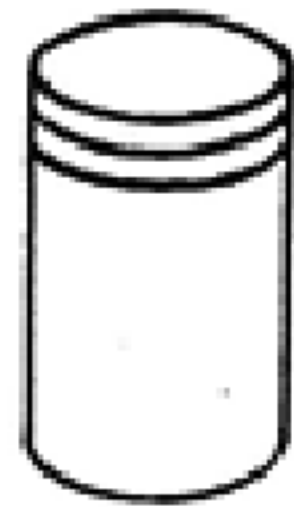
Study the old system of an imaginary Gas Board which went wrong too often and is being replaced by a computer system. The gas-man, who has a book for each particular district, calls at the house and makes a note on the page for the house of the meter reading and the date. If he cannot get in, he makes a note of this and an " estimation procedure " is used for the bill— see later. When he finishes the district, maybe a week later, he sends the book to the accounts office. Here, a large space-consuming card-file system contains a card for each house and on the card is recorded :—the occupier's name and address, the Gas Tariff code for that house,—this is a technical term for methods of paying for gas used. For instance Tariff 1 may be 10p for each therm used, Tariff 2 a charge of £2.50 paid whether or not any gas is used plus 7p per therm used, Tariff 3 a charge of £4 plus 6p per therm used, etc. Obviously, the more gas you use, the more economical it is to be on a higher Tariff number—the meter readings and dates for the last 6 quarters (three months), and finally any arrears from the last bill. This card is taken from the file and the last reading subtracted from the new one (possible error here), the bill is calculated (correctly?), any arrears are added (no comment) an account is typed out (!) and posted. The " estimation procedure " simply means looking up the amount used for the corresponding quarter last year and using this to calculate the bill. The new reading, date and amount are entered on the file, the first of the six previous entries is erased and the card replaced. When the bill is paid, the amount is credited on the card ; if it is not paid within a month, the files are searched and reminders sent out, followed by a demand note. In all, a very complicated and lengthy process which could (and does) go wrong at any of its stages!

For our new system, we require a machine which can " read " written figures (a UNIVERSAL DOCUMENT READER—U.D.R.), a computer, a printer and storage facilities. We shall use magnetic discs as they allow fast access to any piece of information. On these discs is stored the same information as on the card plus a reference number which precedes each address. The gas-man, a computer operator and a special card devised for the meter reading (shown at the top of the chart) are the only ingredients of the system. These cards are filled in, sent in at the end of the day, put in the U.D.R. by the operator and the system takes over. It is self-explanatory from the chart and eliminates all human error.

Why not make up a flowchart for an everyday procedure (or anything else that you feel is of interest, but please not GETTING DRESSED). The editor will be pleased to send a book token to the author of any that we use. If you find any problems with a particular flowchart, send it to the Editor and if we find any interesting problems that we can publish we will be only too pleased to send a book token in appreciation.
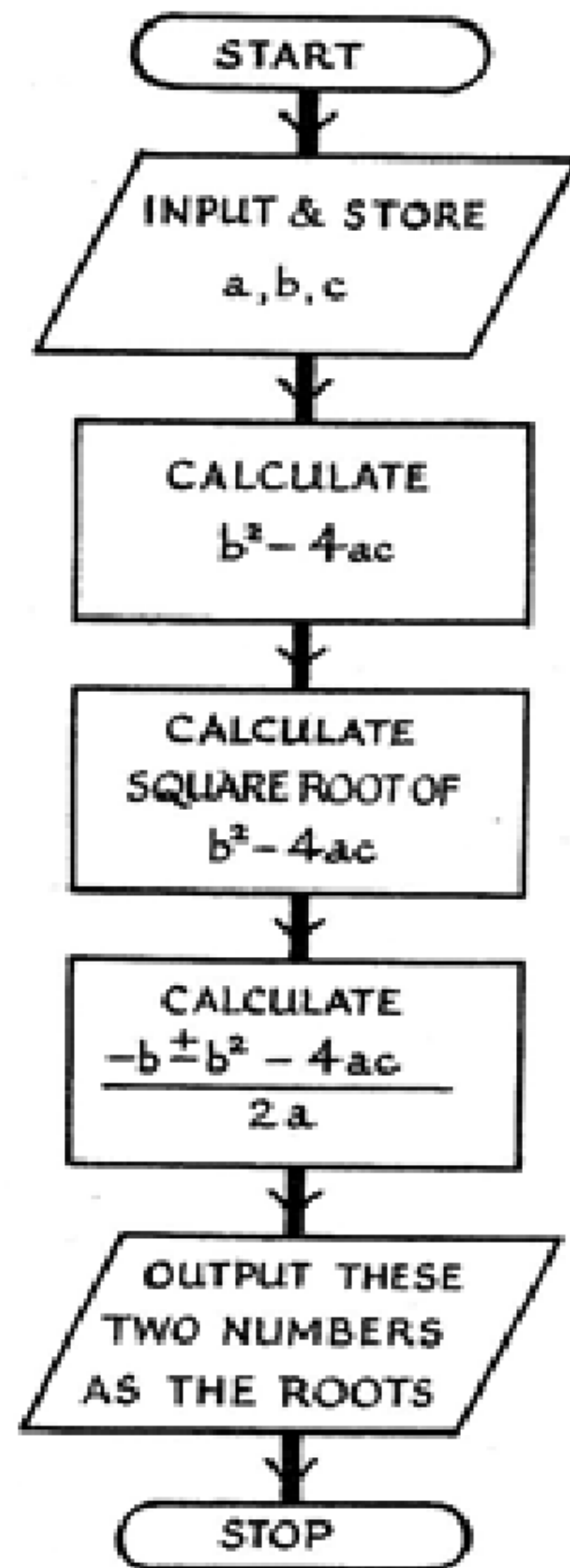
On the left is another flowchart with an error—one step is missing— can you find the error?

P.J.G.

METER CARD

NAME
ADDRESS

← REF. No.
(Previously printed)
← DATE
← METER READING
(Left blank
if no access)

MASTER FILE

INFORMATION FROM U.D.R.

PRINT OUT
NEW CARD
TO BE FILLED IN

IS READING > THAT FOR LAST QR. ?

NO

IS CARD BLANK ?

NO

QUERY FILE
(Print out each day)

YES

CALCULATE BILL USING ESTIMATION PROCEDURE

YES

CALCULATE BILL

PRINT ACCOUNT & POST TO CUSTOMER

COPY OF NEW MASTER FILE

(Old File & new date, new reading, debit.)

BILL PAID ?

YES

ALTER DEBIT TO ZERO

NO

PRINT OUT CUSTOMER'S ADDRESS, AMOUNT OWING & POST TO CUSTOMER