THE PI-PIED PIPER

A chance remark made to the editor after a lecture he had given to the Sheffield branch of The Mathematical Association in the summer term of 1950 started it all. The lecture was on teaching aids in mathematics and included a notice board that hung in the corridor, displaying six or seven items, problems, historical details, etc., all connected with mathematics. A bank of items enabled the editor to change the display each month and a teacher enquired if any thought had been given to publishing the material.

On his return to Leicester, your editor investigated the idea and found that for about £10 he could have 2,000 copies of a single sheet (four pages) of a selected number of items. During the Summer vacation he decided to risk this sum (quite a bit in the days before inflation), and having received the copies, sent bundles of them, together with a circular and order form, to a wide circle of addresses (mainly branches of The Mathematical Association, especially Sheffield, and such schools as were thought to be interested), and the price of the issue No. 1 was the princely sum of one penny (old coinage)!

The response was completely unexpected. We had to return to the printer time and time again until 18,000 copies were sold. As the orders rolled in the problems of handling money, keeping accounts, dispatch, etc., had to be faced, as well as the preparation of issue No. 2. In view of the remarkable response it was decided to increase the size to eight pages and make the price 2d. (twopence old coinage). Some 25,000 copies were printed and all quite quickly sold. Thereafter, as each new issue appeared the circulation figures shot up and the price level became more easy to maintain. At one time the figure reached 130,000 each term, and despite increases of printing costs and postage the 2d. selling price was held until recently. Since these giddy heights, the price of paper, printing and postage have been so enormous that it has meant higher prices for the copies if the venture was to run without loss, and this in turn has influenced our readers' reactions to purchasing copies. Mathematical Pie is registered as a Charitable Trust and does not aim to make a profit but merely to balance its books, and a great deal of unpaid work is put in by a large number of volunteers doing it in their spare time because of their interest in and love for the teaching of mathematics.

The name of the periodical was another happy but accidental choice. Your editor was cycling home for lunch pondering on the problem, trying out a number of possibilities, when suddenly he muttered the word "PHI". However, as he was also thinking of food he spelt it "PIE", and so in order to make it clear that it was a mathematical magazine it became "MATHEMATICAL PIE". He nearly fell off his bike with excitement! Our little emblem "the pi-pied piper", which used to appear at the top of each issue, was also born out of our desire to make mathematics appeal; we produced a series of posters in order that pupils would know that a new issue had arrived and the first of these posters had on it "The pipied piper; get your copy now".

We have all had a lot of fun and satisfaction producing and dispatching over the last 25 years, and we hope that our efforts are deemed to have made a useful contribution to the teaching and attitudes towards mathematics in the classroom. We can only hope that despite enforced price rises the circulation will remain sufficiently high enough to make it possible to continue appearing for the next 25 years.
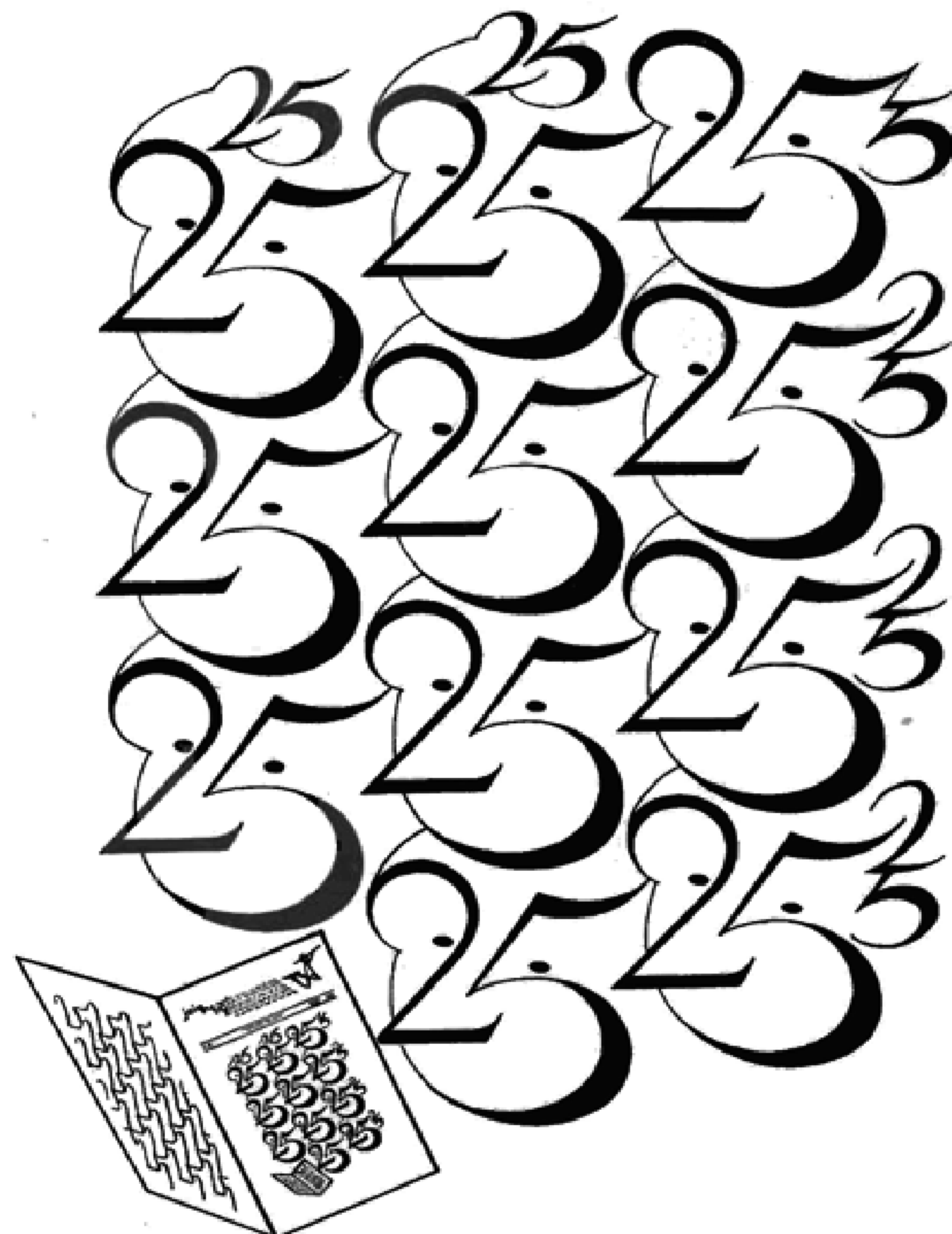
CAT'S CRADLE
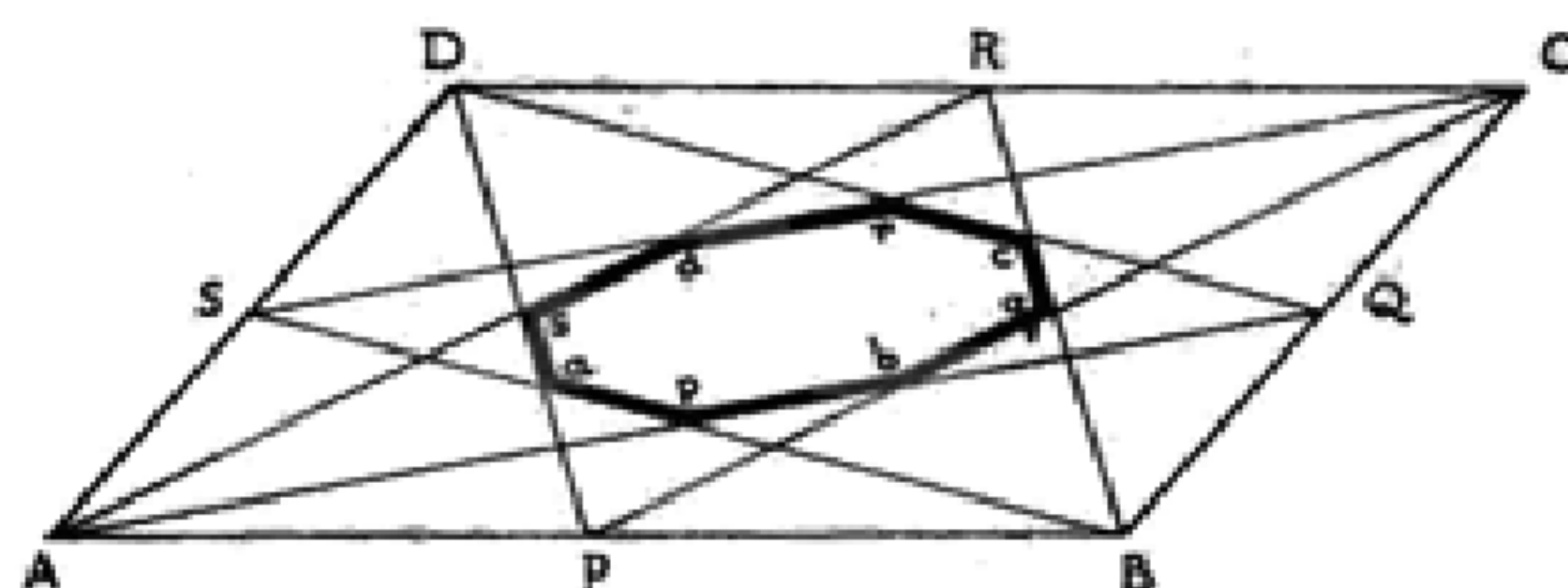
PQRS are the mid-points of the sides of the parallelogram ABCD. What fraction of the area of the parallelogram ABCD is the octagon apbqc rds?

R.M.S.

ODD NUMBERS

There is something rather odd about the following number:—

ONE MORE THAN THE LARGEST NUMBER THAT CAN BE EXPRESSED BY SIXTY LETTERS.

'Extracting a Square Root'

THE ROOT OF THE PROBLEM

Find two right-angled triangles with integral sides whose areas are numerically equivalent to their perimeters.

If the base is "a" units and the height "b" units, can you find a formula which might express "a" in terms of "b"? Are there any other such right-angled "integral" triangles?

Does your formula work for non-integral sides? If not for all such numbers, can you explain?
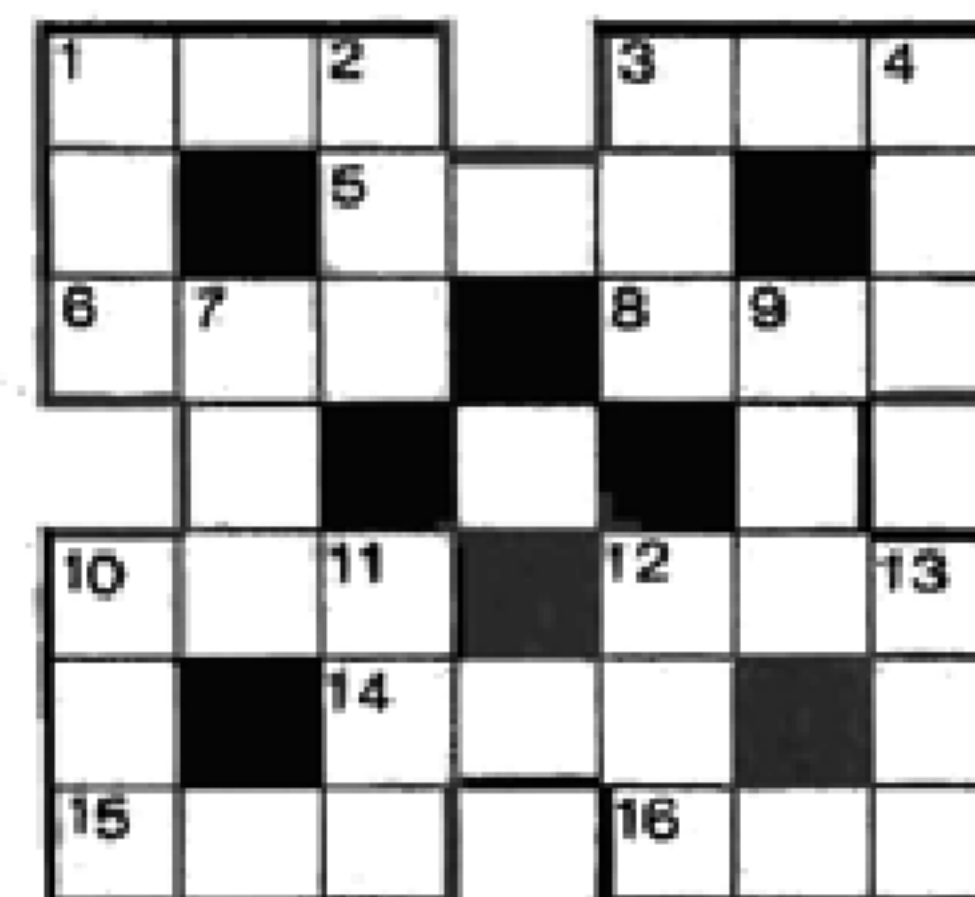
E.G.

CHARLIE COOK AGAIN

Charlie Cook found this problem in his Maths book. "Because of the current I can travel at 3 m.p.h. upstream but 6 m.p.h. downstream. How far away is a place to which the return journey requires 2 hours?"

Charlie's solution was: Multiply 2 hours by 6 m.p.h. and then divide by 3 m.p.h. So the distance is 4 miles.

Oddly enough this is the correct answer. There must be some relation between the two speeds for Charlie's method to give the correct answer. What is the relation?

C.V.G.

JUNIOR CROSS FIGURE No. 67

12. n^3 followed by n^2 where n is a whole number.
14. Six times the next prime above 47.
15. The cost of 7 articles at 39p and 1 article at 19p.
16. (a,b,c) if $2a + 7 = 25$, $3b - a = 9$ and $3c = a + b$.

CLUES DOWN

1. The number of degrees in seven-eighths of a complete turn.
2. 12·25 degrees written in minutes.
3. The perimeter of a square whose area is 1156 sq. units.
4. The value of $2x^2 - 2$ when $x = 10$.
7. $11298 \div 21$.
9. $37 \times 7 \times 3$.
10. The sum of all the even numbers less than 43.
11. The sum of all the perfect cubes between 10 and 300.
12. The area of a square whose perimeter is 68 units.
13. $6pqr + 4r - (5p + 2q)$ if $p = 3$, $q = 5$ and $r = 10$.

E.G.

CLUES ACROSS

1. The prime factors of 105, written in ascending order.
3. The square root of 14 641.
5. Seven cubed.
6. $3 \times 5 \times 37$.
8. Three consecutive numbers whose sum is 21.
10. The larger of the two perfect squares between 400 and 500.

SOLUTIONS TO PROBLEMS IN ISSUE No. 75

HOWE?

There is no limit to the size of the square but the height increases with the length of the side.

FUN WITH FIGURES

The number is 10144927536231884405797. It can be found by starting with 7 and dividing by 7 adding the figures of the answer as you go along.

SPOT THE ERROR

It is correct to the step $-x = -1$, then it should read either $x = 1$ or $x^2 + x + 1 = 0$ and it is the second possibility that was given in the beginning.

HOW DOTH THE BUSY PUZZLER?
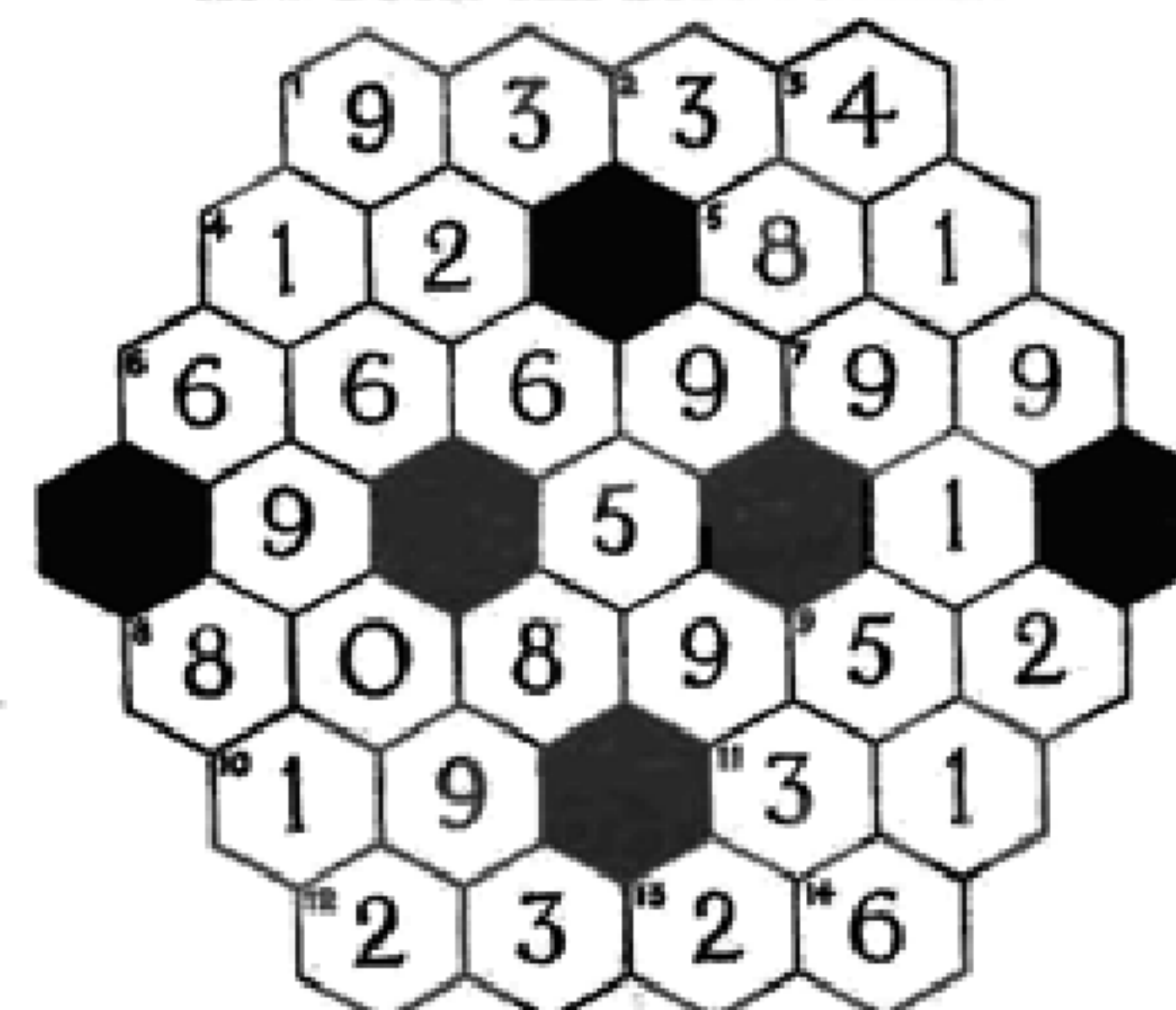
JUNIOR CROSS FIGURE No. 66

PACK 'EM IN—Arrange the cigarettes in alternate rows of 20 and 19 and an extra row can be fitted in giving ($5 \times 20 + 4 \times 49$) = 176, or alternatively the cigarettes could be shredded. B.A.

Stages of a Plane Table Survey.

1. A sheet of paper is fastened to a plane table and a straight line AB is represented to scale.

2. With an Alidade (Sighting rule) sightings from A are taken on P and Q and lines are drawn on the paper.
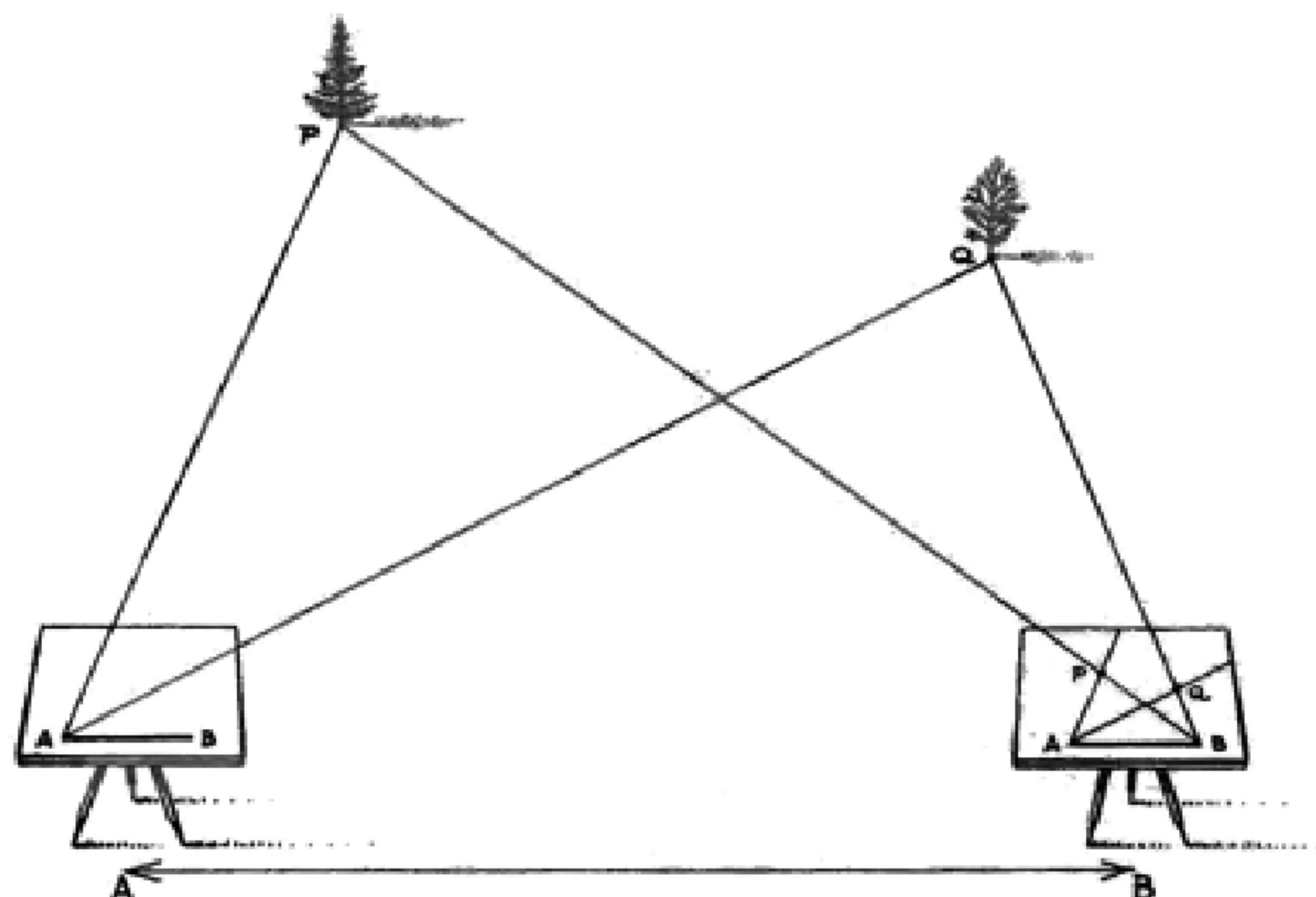
3. The plane table is then orientated at B and sightings from B are taken on P and Q. Intersection of the two sets of lines gives the positions of P and Q.

D.I.B.

MORE POWERFUL

In issue No. 66 we asked for solutions to the problem $a^3+b^3+c^3 = 100a+10b+c$. We also asked if there were any two digit numbers for which $a^2+b^2=10a+b$, four digit numbers for which $a^4+b^4+c^4+d^4=1000a+100b+10c+d$. We have received a letter from Monty Solomon of New York in which he quotes the following results obtained by Mark Fienstein, Andy Sipka and himself using a computer.

1 digit number	0,1,2,3,4,5,6,7,8,9
2 digit number	no solutions
3 digit number	153, 370, 371, 407
4 digit number	1634, 8208, 9474
5 digit number	54 748, 92 727, 93 084
6 digit number	548 834
7 digit number	1 741 725, 4 210 818, 9 800 817, 9 926 315

Our thanks to Monty and his two friends.

R.M.S.

SENIOR "DOUBLE CROSS"

Every digit in the cross figure left, matches with a particular letter in the crossword, right. Cross-references may be needed to help with the solution. Finally, decode the rhyme.

CLUES ACROSS

1. Five terms of A.P. whose common difference is 2 and whose sum is 25.

5. Angle AXJ if polygon ABCDEFGHIJ is regular and X belongs to the set BCDEFGHI.

6. $\sqrt{8}+\sqrt{7}+\sqrt{6}-\frac{1}{50}-0.04$.

8. ADD+ODD−ODE.

10. x^2+x+8 if x is the fourth root of 14 641.

11. (a,b) given $a>b>0$, $8ab=468$ and $4(a^2+b^2)=493$.

12. EA.

14. (b,a) see 11 across.

16. The co-ordinates of the two points on $y=x^3-15x^2+71x-96$ where the slope is 8 (Larger value of x first).

18. p^3-q^3+10 if $2p+3q=17$ and $3p-q=9$.

19. (x,y,z) if $x+y+z=22$, $x+y-2z=4$ and $5x-5y+3z=10$.

CLUES DOWN

1. Product of three single-digit odd numbers.

2. (u,v) if $uv=35$ and $\frac{u}{v}=1.4$.

3. Subtract half a dozen dozen bakers dozens from a quarter dozen dozen of cubed bakers dozens.

4. 2nd, 3rd, 4th terms of $t_r=4t_{r-1}-10(6-r)$ given that $t_6=24$.

5. $n^2(n-4)$ and $m^2(n-4)$ if $n+m=10$ and $m+4=n$.

7. $3^3+2(12345+2468+369+48+5)$.

9. Add in base 12: $54576+24978$.

13. (S^2, S).

15. $(\frac{1}{4}R^3, R)$.

17. $2S^2$.

CLUES ACROSS

1. Not the most we could steal.
5. Look! It's the first half.
6. Three down rays can make you so red it's uncomfortable.
8. Units end impersonally and possessively.
10. Usually on the top, but for a slide it's in the middle.
11. Reverse black skill and sing nonsense, la!
12. Fifty from the pile of magazines.
14. Something caught in a trap, not quietly.
16. "Let it stand". When son follows, it's a hat.
18. The central idea of "exists".
19. Look for this poor indication that an animal passed by.

CLUES DOWN

1. A great deal may be under the hammer.
2. Cleopatra's "worm"—but not quiet, as you see.
3. Is so large an amount of free energy available.
4. A prefix of three letters (very appropriately).
5. "S" sticks in his lips, as he does this.
7. Strides back and removes two letters—or even cuts out some words.
9. I'm inclined to think this is a bit outrageous.
13. It's here again—see 8 across.
15. Tuneful one of the ancient "elements".
17. How find it? Point out toe!   E.G.

SENIOR "DOUBLE CROSS" RHYME

7519 735 7541867, 49 49 9810, 2367479 032637730 941 243 47 7810: 0356 6350367, 47 49 143 86 6351? 08 7246497 7856 98 2437 522351?

SURVEYING

An Early Egyptian Level

Egyptian Hand-held GROMA – for constructing right-angles

c.3000 B.C. Egyptian surveyors possibly formed right-angles by stretching a knotted rope divided in ratio: 3:4:5

THEODOLITE invented by Digges 1571

Between 150 B.C. and 250 A.D. Heron's Levelling Rod (Greek)

Pulley

Sight line

Heron's DIOPTRA

$$\frac{h}{x} = \frac{CP}{BA}$$

ASTROLABE – probably invented by Babylonians but not developed as a surveying instrument until 1000 A.D. by the Muslims

16th Century Military Drumhead Trigonometry

Used for modern land measurement, the 20 metre – chain has 100 links. It is derived from Edmund Gunter's 66ft. chain of 1620

STAGES IN UNDERTAKING A CHAIN TRIANGULATION :–

② "Follower" lines up "leader" with distant ranging pole. Leader marks each chain length with a steel arrow from a set of ten. As each chain length is completed follower collects arrow as a tally

① Examination of ground and establishment of stations (A,B,C,D,etc.,) to provide a strong triangular network

straight edge and plumb line

Hill top beacons directed the Roman road builders

MERKHET Earliest recorded sighting instrument (Egyptian)

(Rib of palm leaf split at widest end)

Roman GROMA Two plumb bobs dipped in water steadied the instrument

cross-piece is equal in length to each division

c.15 B.C. Vitruvius's CHOROBATES (Roman) Filled with water, the trough was used as a level in windy conditions

16th Century Cross-Staff for determining inaccessible lengths

sensitive level viewed in mirror

Tilt box level

Tilting Screw

Modern Metric Staff and Dumpy Level

A MODERN SYSTEM FOR MEASURING AND RECORDING LEVELS (Metres)

Back Sight	Intermediate Sight	Fore Sight	Rise	Fall	Reduced Level	Remarks
1·30					50·00	T.B.M on gate post
3·40			0·80	0·40		
	2·50			0·90	49·20	T.B.M on bridge
	4·60			2·10	50·00	

Temporary Bench Mark (assumed value)

Field book showing method of recording detail from station A to station C. When lines AB, BC, CD and DA, together with a check line BD, have been booked, a map can be drawn to a convenient scale (All units are in metres)

11·8 m. from A
8 m. from A

③ An off-set being taken from the track at right-angles to the chain line. Dotted lines represent tie-lines from gate-post to chain

Smith

Man probably had a crude appreciation of land measurement as far back in history as the Neolithic period when the discovery of agriculture enabled more settled communities to develop. The oldest records of surveying instruments and techniques date from the early Egyptians: the Rhind papyrus (c. 1500 B.C.) refers to the harpedonaptae, or rope stretchers of the time. Construction of the pyramids demanded accurate measurement and levelling from the surveyors available. Incorporating a split palm leaf and a plumb bob, the merchet was used for the orientation of pyramids and temples by star observation.
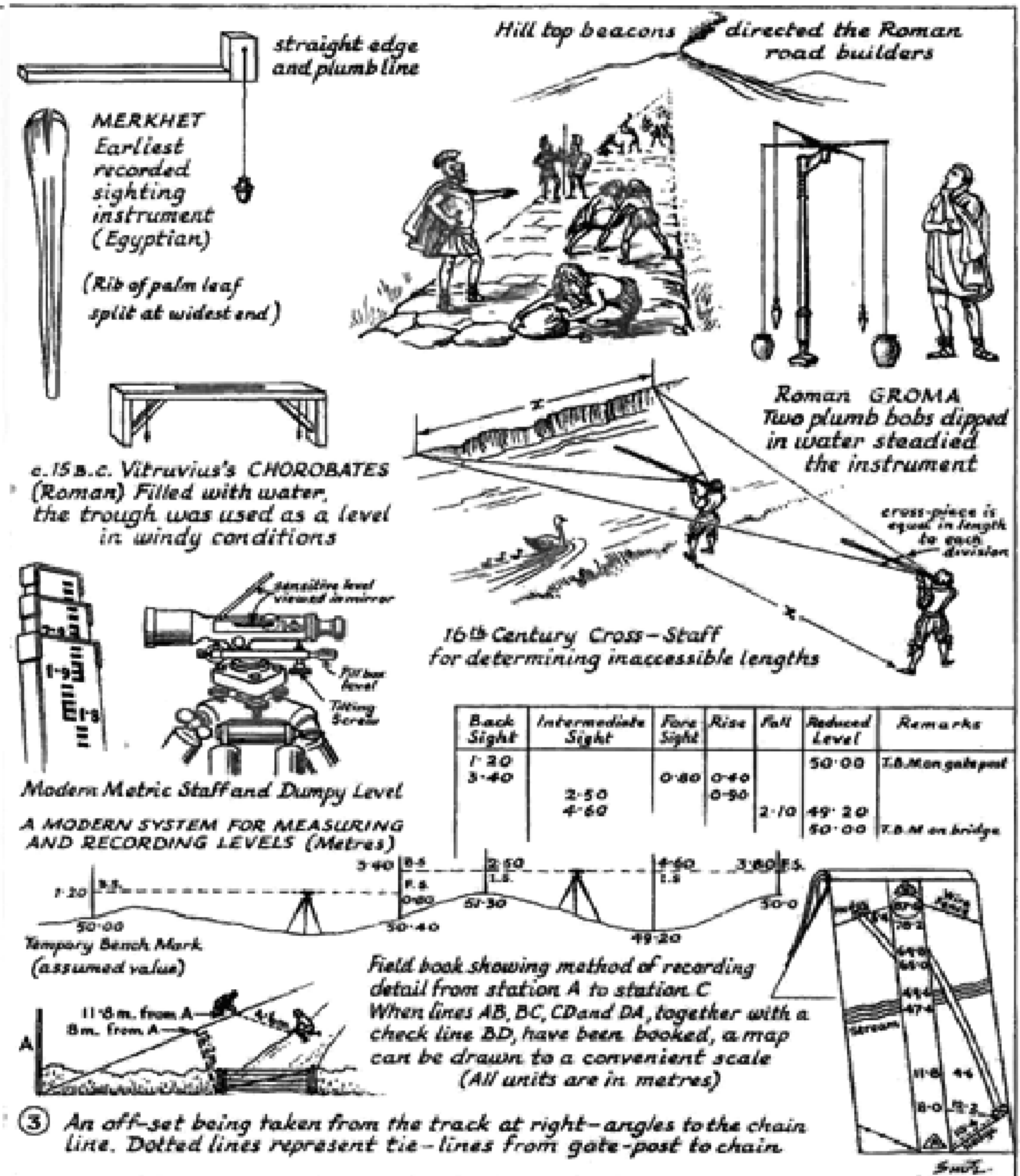
Unlike the other mathematicians of Ancient Greece, Heron of Alexandria was particularly interested in the practical applications of his subject. In many respects, he was the Father of surveying, inventing instruments which, in principle, differ little from many of those employed by surveyors today.

Ref.: History of Mathematics, Vol. II, by D. E. Smith; Surveying Instruments, Their History and Classroom Use, by E. R. Kiely; Encyclopaedia Britannica; Mathematical Pie, issues No. 58 and 63.